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ABSTRACT

In an effort to prepare students for the 21st century, this book provides ideas and suggestions for educators on critical areas in mathematics education. A series of articles is presented on content changes, instructional strategies, and the role of computers. These articles include: (1) "Status of Computers" (John Ellsworth); (2) "New Topics for the Elementary School Math Curriculum" (Tamas Varga); (3) "Mathematics for the Twenty-First Century" (A. I. Weinzweig); (4) "The Teacher and Curricular Change" (Fred D. Carver); (5) "An Innovation in Assessment: The MANS Test" (Martin Herbert); (6) "On Assessment" (James L. MacLean); (7) "Five Anchors" (T. C. O'Brien); and (8) "Computers and the Future" (Robert S. Hoeke). Lists of suggested readings are provided for parents and teachers and for background and research. Catalogs and periodicals are also identified. (ML)

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TOWARD THE 21ST CENTURY IN MATHEMATICS EDUCATION

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Toward the 21st Century in Mathematics Education

Teachers' Center Project

SIUE

Southern Illinois University
at Edwardsville

INTRODUCTION

This project had its origins in a conversation I had with a theologian. "You would marvel at the work that's being done in our field," he said. "There have been some really great advancements. The unfortunate thing is that it'll take fifty or one hundred years for the new ideas to dribble down to the local churches."

What about education, I thought. How long does it take new ideas to become commonplace?

Present day schoolchildren will spend most of their lives in the Twenty-First Century. And some of their children will be alive in the Twenty-Second Century. Yet in terms of content and ways of bringing about learning we are mired in the practices of the Middle Ages.

In mathematics education, our interest here, it takes eons for new ideas to find their way into the educational delivery system. At the elementary school level, mathematics education consists of transmission of facts, rules, procedures, nomenclature and notation -- virtually all of it concerned with speed and accuracy at arithmetic computation. Is this a good fit with the requirements of life in the Twenty-First Century? The answer is a clear-cut NO.

A large number of teachers and parents are unhappy with what's commonplace and want some seedlings for thought and development. What is it that we could/should be doing in mathematics education to prepare children for the Twenty-First Century?

This book is a first attempt to answer the question. It is also an attempt to enlist and enlarge the "delivery system" the Teachers' Center Project has developed during the past ten years. Rather than swallow ideas from on high, as is commonplace elsewhere, Teachers' Center folks are encouraged to take what's here and bend it and extend it. For that reason, a lengthy bibliography is given at the end of the book and authors' addresses are provided for further discussion, argument, elaboration, growth.

In sketching out this book, we tried to attend to possible content changes, to approaches to bringing about learning, to assessment of learning, and to the role of computers.

Warmest thanks to the participants in our December seminar at SIUE: Jim MacLean, Izzie Weinzweig, Bob Hoeke, Tamas Varga and Shirley Casey. And warmest thanks to the authors of the papers as the seminar fleshed out into the present work. Best of all, warmest thanks to the teachers and parents whose foresight and quest called for this book in the first place. The ball is in your court now. Knowledge is not a spectator sport.

T.C. O'Brien
Thomas C. O'Brien

Edwardsville, Illinois
26 March 82

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THE STATUS OF COMPUTERS

John Ellsworth

THOMAS C. O'BRIEN: I am happy to introduce Dr. John Ellsworth, President of Centarus Computers. John has been a professor in the government department at Southern Illinois University until recently and now he is in the computer field full-time. Some primitive questions first, John. What is a computer?

JOHN ELLSWORTH: A computer is really a very rapid and stupid calculating machine. It consists of a large number of switches which can be either on or off.

O'BRIEN: What does a computer do?

ELLSWORTH: Basically a computer can add, subtract, and compare values. It's very fast at doing this and because so many addition and subtraction and comparison operations are possible in seconds, it appears to be almost magical to us. It's really not. It's really a very simple device. Its operations are all setting switches to on or off values.

O'BRIEN: That's all it is?

ELLSWORTH: That's all it is.

O'BRIEN: One of the things that Americans have become aware of in the past two or three years is the computer as part of one's everyday life. It used to be that there was one computer at the University, it took up a large room which had to have special air conditioning, and so forth. Now the computer is pretty widely known. What accounts for the popularity of the computers?

ELLSWORTH: Obviously one thing that makes the computer popular is that now the ordinary person can afford one. It has become a relatively inexpensive device. The computer used to be the exclusive property of large corporations or large government agencies or a major university because of its size, its need for a special environment, the tremendous initial investment, and the need for specialized personnel to run things.

The critical breakthrough that made the computer less expensive was the invention of the integrated circuit or silicon chip. This has allowed us to miniaturize computers and to put many thousands of circuits on one very small micro-chip. The on/off switches I was talking about are really a series of circuits. Thousands of circuits now can be condensed into a very small space and because of the characteristics of the silicon chip, we don't need a special environment. In general, if you're comfortable in an environment, the computer will also be comfortable.

O'BRIEN: How big is a chip?

ELLSWORTH: A chip can be as small as a third of the size of the nail on your little finger.

O'BRIEN: Those little chips do the work that the big behemoth machines used to do in the 1950's and 60's?

ELLSWORTH: Essentially that's correct. In fact, small computers like the one that's sitting on my desk have in many ways more capabilities than large computers had 10-15 years ago. They have more memory storage. They can perform their operations and calculations more rapidly. Of course, the large computers have taken advantage of the integrated circuit and have become much more capable than older style machines which had iron core memories and, even earlier than that, vacuum tubes.

O'BRIEN: What's a ballpark figure for a computer these days?

ELLSWORTH: Between $300 and $400 for a home computer. For a serious home application the range is $2,000-$3,000. For a small business the range would be $3,000-$5,000 and up. These are machines which would have cost hundreds of thousands of dollars not so long ago.

O'BRIEN: But the fact that computers are cheap doesn't alone account for their popularity. Snow is cheap and people are not running to buy snow.

ELLSWORTH: That's absolutely right. The machinery by itself is absolutely useless. One of the things that we're really concerned with is to educate the public to realize that the machinery is only useful when software (i.e. programs or instructions) makes it perform some useful task.

Much of the popularity of computers in terms of the hobbyist and the home computer user has tended to spring from the entertainment value of the device. The ability of the computer to play games with you, or allow you to do graphics, or to make music -- this is lovely entertainment. Becoming more and more popular is the ability of the computer to talk to you rather than to send messages to you on a printer or on a screen. These are features that make the computer seem more human.

The arcade games that people put quarters into in large numbers have attracted a large following. These arcade games have migrated over to home computers, and a lot of the younger people are drawn to the computer simply because it will do the kind of things that the game machines in the entertainment arcades will do. I suppose that that's one of the things that makes a computer popular. They produce sound effects and pictures and test your hand-eye coordination and compete with you in some kind of a game.

Logic games are in another class of entertainment. One of the characteristics of the computers which I just touched on very lightly is its ability to test or to compare values. Logic can be programmed into the computer and so games using logical rules easily become part of the repertoire of the computer. For years programmers worked to get big computers to play a good game of chess. Chess migrated to personal computers almost as soon as they went on the market. A very successful software company called Personal Software made its initial fortune on the sale of a popular chess program for small computers.

The fact that computers can perform complex logical operations through a set of instructions that respond to the moves that you make with pre-programmed logic, challenges people first to beat the program. Then, once a player is beating the machine he or she may try to see if they can write a better program, one that would be more challenging. So in two ways the computer chess game engages one mentally — testing one's playing skill, and inviting one to write a better program.

Today there are all sorts of card games, strategy games and fantasy games that incorporate both logic and imagination. All of this occurs at the entertainment level without getting into any serious business applications or educational applications.

O'BRIEN: Even at the game level, a great deal of thinking and strategy-making and learning takes place on the part of kids.

ELLSWORTH: Yes, it's very sneaky.

O'BRIEN: Sneaky and painless.

ELLSWORTH: One of the first things that we had on our home computer was a game called "Ad-libs". That's a popular party game that people have played for years. You have a little story and various people in the room are asked to supply nouns, verbs, adjectives, adverbs which are inserted into blanks in the story. It makes an amusing exercise because when they're read back, the words don't necessarily fit in a normal way in the story.

When we had our first home computer, my daughter's friends would come over just like it was the first television set on the block. They would play "Ad-libs" and they would take turns supplying the words as they were asked for. Now some of these young people didn't know their parts of speech very well. The computer would say, "Give me an adverb." One of the kids would say, "What is an adverb?" And the other kids would respond, "Well, you dummy, an adverb is such-and-such."

So the kids taught each other the parts of speech in order to play the game. They didn't realize that they were getting grammar lessons. If you had told them to come over to our house in order to learn grammar, they would have stayed away in droves. But because it was fun, and because there was peer pressure, the kids who were engaged in the game were, in a very unobtrusive way, learning parts of the speech.

O'BRIEN: *So unofficial learning is a major part of home computers. What everyday household uses have you ran across in your work?*

ELLSWORTH: Well, the simplest applications are those that keep track of your checkbook, or keep your tax records. These are the first things that people think of. Computers keep track of money. (After all, computers have been sending our credit card bills and our bank statements for a long time now.) So if you had a computer you could keep track of your money better. Another thing computers are very good at is storing information and retrieving it in different forms. So people keep household inventories and shopping lists. They also use computers to keep track of stocks and their current values and even to keep track of their recipes. The computer stores anything that you would keep track of -- Christmas card mailings, family schedules, or anything on that order. Such housekeeping chores in terms of money and items and dates and phone numbers are common uses for the computer in a home.

I think the people who became enchanted early with the computer viewed it much as people in the past viewed television sets or ham radios. However, with the television set one is a consumer because you don't have to know a lot to be able to tune your set. It's something that you can play with. It's a passive diversion. On the other hand, you have ham radios which require more active interaction. Ham operators started early hooking small computers into their radios to encode English or any language into Morse code and to receive Morse code messages and decode them.

As with the ham radios, much of the really exciting computer software and many of the exciting developments come from people who were simply interested in seeing if they could make a computer do something. For instance, there is a growing interest in using the computer in conjunction with the telephone to transmit programs to other computers and to send information back and forth between computer owners and large scale information utilities. Large scale information utilities do everything from providing news stories on just the topic you want, to listing stock and commodity prices, to making travel reservations, to finding bargains in shops and services, to searching employment advertisements, to just anything that involves managing or exchanging information, even exchanging money. It's coming to be that with a computer and a telephone, a person can reach around the world.

O'BRIEN: A person sitting in one's living room can be in touch with information banks all over the world?

ELLSWORTH: All over the world. Right now most of the information services are specific to a country. Great Britain has a national service and there are a couple of large utilities in the United States. The Source is one that our company markets.

O'BRIEN: I'm an ordinary layman. I live in St. Louis and I'm interested in baseball. I like to read. I've got three kids. How would I use all this?

ELLSWORTH: You would take your computer and simply dial up another computer. This computer could be a large computer like The Source (which has tons of information in it) or it could be a kind of underground, informal communication -- sometimes called a Networks station. Such a station could be a computer in a store or a computer in someone's home. One of our customers has one in his home. He's a school teacher and he leaves his computer "Networks" on during the day and lets people tap into his information service and leave and pick up mail.

O'BRIEN: By mail, you mean electronic mail?

ELLSWORTH: Electronic mail. It could be a simple message such as, "Hey, Joe, do you want to go out Friday night?" or it could be a program that someone has written that he or she wants to share.

O'BRIEN: So in a sense whatever anyone is doing on a computer can become available to my computer by telephone.

ELLSWORTH: That's right, as long as your computer will run their program.

O'BRIEN: Much of this is done on a free-lance basis, so it doesn't cost any more than a telephone call? A lot of informal knowledge which is available to my household just by phoning up?

ELLSWORTH: Right, In many cases, it has nothing to do with a commercial enterprise. It's just a lot of people who are sharing their hobby, using the computer and the telephone as tools.

O'BRIEN: There are a lot of programs available in the public domain? You don't have to buy every bit of software?

ELLSWORTH: That's right. You can get into a users' group (as they are called), which is essentially a group of people who use computers or are interested in them. Also you can get into the telephone utility thing. And often if you simply find the right person to contact, people will supply a program free. There is also, of course, a large and growing body of commercially available software at anything from $5 to several thousand dollars for a program.

O'BRIEN: And this ranges from game playing to income tax keeping to educational programs in school subjects?

ELLSWORTH: Yes, everything from what used to be big time computer applications for businesses to playing checkers against a skillful player.

O'BRIEN: You said earlier that people get tired of amusement programs and then enter the program to hike things up a bit. I can beat the chess player, now I'm going to enter the program and alter it so that it gives me a stiffer task. What abilities are called for here? I'm not talking about the commercial marketplace. I'm not talking about a career as a computer programmer. For the layman programmer, what abilities are needed?

ELLSWORTH: Well, the first thing that he or she will have to do is to learn some sort of programming commands, something about the programming language. The most popular language is called BASIC, which is a relatively easy-to-use programming language. The computer can only respond to a set of strictly outlined logical instructions, so you have to learn to be logical; the computer will do exactly what you tell it to do. You must program your instructions according to its rules, but you can put in as much imagination as those rules will accommodate.

The first thing is that you have to be able to think logically. The typical pattern for a computer user is that one begins by writing simple programs like printing your name on the screen, or adding two numbers, or listing numbers from 1 to 100. From there they go on to learn more complicated operations with a programming language. At some point, some users become frustrated with the limitations of the language, and begin to want to understand how the programming language itself addresses those computer switches we were talking about. That is, they begin to want to know something about how the machine operates electronically. This requires that they get into machine language. Unlike higher level languages, machine language does not look at all like English. It involves much more abstract symbolic and mathematical logic. The people who have become very creative and commercially successful have typically become very proficient at speaking the machine's language and not being restricted by a higher level programming language.

O'BRIEN: But higher level programming language such as BASIC is one giant first step?

ELLSWORTH: Yes, and that's where most of us stay. We don't get down into the guts of the machine. In fact, for most of us, the higher level languages are so powerful that virtually anything we want to do can be done without knowing very much about how the machine operates.

O'BRIEN: When you deal with the layman who's going to address the computer other than as a mere consumer, what difficulties do you see as people learn to program?

ELLSWORTH: Well, I think the initial difficulty that we find is fear. There's a love/hate affair with the computer. There's a fascination with it and also a fear because it's not understood. That fear stands in the way of most older people. It does not inhibit children at all. By and large, children take to computers immediately. We have to monitor the visits of children to computer stores because they come in in droves. Some come in to be entertained. Some of them come in and want to learn to program. It's much the same way our generation was with automobiles.

O'BRIEN: They like to tinker.

ELLSWORTH: They're fascinated. Neither of my grandmothers ever learned to drive a car, but their grandchildren couldn't wait to drive. It's very much the same way now with computers. Some of us adults are very turned on and others are afraid. Many, many children are turned on.

I think the key to overcoming our fear is to perceive that the computer is going to do something for you. It may be something for business or something for a profession or something for education or something that is just entertaining.

O'BRIEN: It will help translate your Latin homework. This I've seen done.

ELLSWORTH: We have some very exciting things developing and nobody knows exactly what shape they will take.

We certainly see some of the future in the products that are now becoming commercially available. One extremely exciting area is computer art. Such an idea can turn you off if you think that art has to be simply a handicraft type of thing, but the use of the computer to experiment with different color patterns and different shadings (either by taking a photographic image or by creating a graphic image) is a very creative area.

The same thing is true with music. In the last ten years or so we've seen the increasing popularity of electronic musical instruments and sound that is modified electronically. The computer can control the synthesizers that do this and musicians are now recording on small computers, editing performances with the computer, changing the pitch of the music, changing the timbre, changing the rhythm — essentially composing and revising music on the computer. Even kids can do this.

O'BRIEN: So you're saying something very important. With most electronic devices -- TV for example -- you are a passive consumer. The computer enables the user to be an innovator.

ELLSWORTH: It's an interactive device. You can interact with it to become a creator, not simply a consumer of someone else's product.

O'BRIEN: Given the availability of the computer, what things are we doing in schools how that might not be so important as they once were?

ELLSWORTH: A lot of boring stuff. Anything that is taught with repetitive drill and practice can be done with a computer. With a little creativity it can be made fun and interesting, even if you simply want to drill people on fundamentals. The fact is that children's skills in such things as arithmetic computation or spelling can be replaced by a computer. This seems to threaten current practices, but those boring repetitive tasks could be assumed by the computer if we wanted to free children to think in terms of more creative and logical and abstract patterns.

Whatever we do, it's clear that we can no longer focus on memorized "fundamental skills" as ends in themselves in the school. I think it's up to us as to whether we do away with the need for these skills, they're certainly not going to be as important as they have been.

O'BRIEN: For example, there are now packages which one attaches to one's typewriter which have spelling vocabularies of 10,000 words.

ELLSWORTH: Well, you can get dictionaries on the computer that check your spelling on anywhere from 5,000 to 100,000 words. That means that you can sit down and compose something and the computer will indicate which words may be incorrectly spelled.

O'BRIEN: But context is an important thing. The word "hair" is different from the word "hare". It's the context that tells me which word to use.

ELLSWORTH: Yes, you still have control. You are left with the decision as to which is correct. It's possible to program an algorithm which would check to see whether you meant "hair" or "hare" but most of the present spelling programs are less sophisticated than that.

O'BRIEN: Context checking is something that is coming?

ELLSWORTH: Yes, that's possible. I was doing it 15 years ago in terms of some of the work I was doing, but it takes a very complicated dictionary.

O'BRIEN: John, you have talked about the nature of computers, their present and future roles in American life, and their role in education. You have clarified a great many issues. Thank you very, very much.

ELLSWORTH: Thank you for the opportunity to make this information known.

NEW TOPICS FOR THE ELEMENTARY SCHOOL MATH CURRICULUM

Tamas Varga

1. Introduction

Is there any virtue in adding new topics to the Elementary School Math Curriculum?

Is it not more important to teach a limited number of time-honoured topics (mainly arithmetic) in a meaningful way, than to cram the curriculum with too much stuff, risking to neglect the essentials?

Since the beginning of the New Math movement, almost a quarter of a century ago, such issues were repeatedly raised, and they are just as actual and relevant as they ever were.

The late Israel Wittenberg never ceased to stress the importance of "how" as compared to "what" in school mathematics until his premature death in the mid-sixties. He was one of the persons inspiring the Memorandum of reputable mathematicians (one of them was George Polya) published in the American Mathematical Monthly in the early sixties. This memorandum defined the extreme views of the reformists, and emphasized continuity.

At about the same time, Hans Freudenthal, also a great mathematician and one of the most influential math educators in the world, raised his voice in defense of traditional values in school mathematics, especially at the elementary level. One of his strongly worded articles published in the Swiss-based international journal L'enseignement Mathematique, bore the title "Teaching Up-to-Date Mathematics or An Up-to-Date Teaching of Mathematics?" (L'enseignement des mathematiques modernes ou l'enseignement moderne des mathematiques?) One of the points he made was that elementary school teachers knew what they were doing when they were teaching, for instance, subtraction in a way that covered all possible kinds of meaningful applications (word problems) leading to subtraction. He included a long list of such word problems each of them about just one specific subtraction of two digit numbers.

Pearla Nesher and J.G. Greeno took up the issue recently. Their interdisciplinary research (semantics, psychology, education and mathematics are involved) throws light on the difficulties encountered by kids at solving simple one-operation word problems about addition and subtraction.

With all that in mind, should we not cheer Samuel Greitzer of Rutgers University when he writes in a letter to Robert B. Davis (published in a recent issue of Journal of Mathematical Behavior) that elementary school mathematics should be concentrated about arithmetic rather than whatever has been offered under the name of New Math? Or should we not join the effort of a world renowned Russian mathematician, Pontryagin, who expresses rather similar views in an article published in a recent issue of the most influential ideological journal "The Communist", blaming Soviet mathematics education because of a departure from its traditional values?

It is, of course, impossible to do justice in this introduction to positions expounded in much detail elsewhere. But it should be made clear that those defending continuity in math education against less than cautious innovations are in certain cases the strongest promoters of sensible innovations. The Dutch IOWO project inspired and led by Hans Fruedenthal is the most conspicuous example. Having in mind projects like that, and scanning through the innovations brought about since the mid- or late fifties, let me reiterate the question: Is there any virtue in adding new topics to the Elementary School Math Curriculum? and formulate my answer this way:

Numbers from natural to real and beyond certainly remain basic components of elementary education. Numeracy is not less important in our culture than literacy. But both of them are and remain important in a rapidly changing and extended sense. Numeracy is not any more the four rules up to long division just as literacy is not any more spelling and pronouncing. The growth which starts with these extends to mathematical and scientific inquiry on the one hand, visual culture and self-expression on the other. Moreover, these "two cultures" are merging. Among other things, school mathematics can become more artistic, more appealing to the emotions.

To some degree there is virtue in continuity. And there is virtue in change. In continuous change.

2. Multiplication and Combinatorics?

A furious father rang me up the other day. He complained about a math problem his son was given as homework in grade five. He saw my name on the workbook and found my number in the directory. The problem was about three groups in a class; nine owls, ten wolves, and eleven giraffes, and one person of each patrol had to be elected to a committee. The boy had to figure out in how many ways this could happen. "This is combinatorics," the father said. "I know, I always got the best marks in mathematics." "Oh, you are right, that's combinatorics indeed," I replied. "But I learned combinatorics when I was 18. Are you serious about bringing down material designed for 18-year olds to grade five?" he asked. "Oh, it does not start in grade five! We do it in grade two," and I told him about problems of getting dolls dressed in all different ways when there is a choice of three caps and two coats. The conversation ended in ten minutes in a much more friendly tone then it had started. He was not so unhappy any more with the idea that a problem for multiplication may have something to do with ideas he learned at a more mature age.

By the way, I must say that I am not particularly happy with that word problem. Selecting one person from each group may arise, but the question about the number of possible selections is less than natural. Maybe we should transform it into a probability problem of this kind: X, Y, and Z are good friends. They are members of the three different clubs. Members of the committee are selected at random from each of the three clubs. What are the chances that precisely those three will be selected as members of the committee? The number 990 hardly is the answer to a question naturally arising in children. Curiously enough, the number 1/990 or "about 0.001" may be more meaningful for them as a rather low level of chance. This tends to be the case, especially if they become familiar with chance situations and numbers like 0.5, 0.1, 0.9, 0.01, 0.99, etc., as probabilities of events are associated with earlier personal experience. The moral of the story for me is that integrated mathematics (multiplication integrated with combinatorics, combinatorics integrated with probability, etc.) is more natural, more meaningful, more appealing than separate disciplines.

3. Building coloured towers and asking the right question

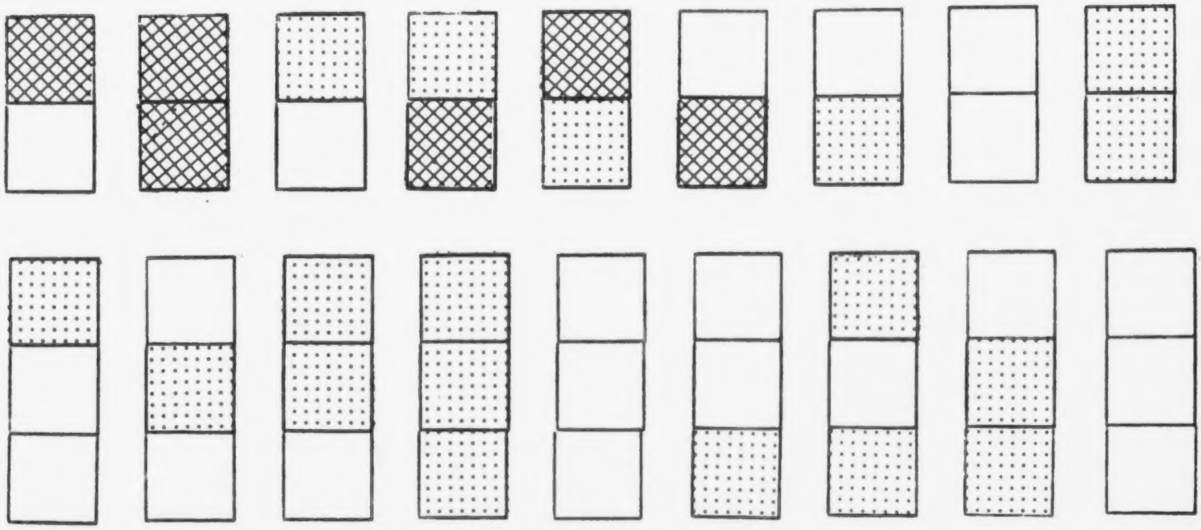
The problem with "in how many ways can you. . ." in the previous section is that it is too adultish for a fifth grader, too much smelling of a textbook. Getting dolls dressed is different for second graders because it is an activity enjoyed by kids: they see the three caps and the two coats, they fit them together, they may even discuss which ones suit better, and this kind of activity becomes a fundamental paradigm of multiplication. The question about the number of possible cases is preceded by other questions or suggestions such as these:

> Choose a cap and a coat and dress the doll with them,
>
> Do it in a different way,
> Do it in still another way,
> Do it in as many different ways as you can.

Only then comes the question about the number. Then they do the same with maybe two caps and three coats or other garments or quite different embodiments of the "direct product" idea.

With numbers as small as in this example, some or even all intermediate questions may be skipped on occasion, depending on the kids' past experience. but the teacher should be aware that they may be needed as stepping stones on other occasions.

An activity at a slightly higher level is this: Two teams of kids build towers of centimeter cubes or other colored building blocks. ("Tower" here means a building you build only upward. It consists of floors, like a house.) The teams have different assignments. One team has blocks of two different colors, and they build three-floor towers. The other team has blocks of three different colors, and they build two-floor towers. At first they build just one, then more, then as many different ones as they can and then comes the question of how many different towers they could build. They they compare the buildings, they discuss the results, maybe they change roles (using other colors this time). There is something unexpected about the result.



The unexpectedness has its roots in the previous type of activity. There it did not make a difference whether there were three caps and two coats or the other way around. Therefore kids (some, at least) would expect that here too there should be a ninth row of the second kind. But whatever they try will be the same as one of the eight they have already built.

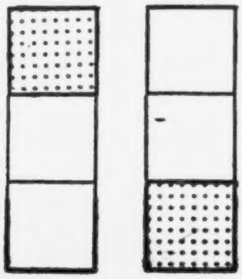
Just as putting clothes on dolls is a fundamental paradigm about multiplication, building towers of colored blocks is a most important paradigm about powers. Whether or not they introduce the notation at this point, what surprises them is the fact that $2^3 \neq 3^2$, which will, much later, be an instance of the non-commutativity of the fifth operation, raising to power.

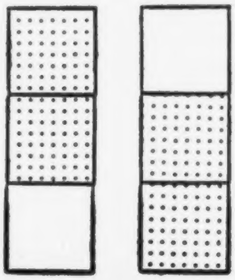
How about two colors and four floors on the one hand, four colors and two floors on the other?

Deep questions of logic are lurking below the surface. No hurry to make them explicit. Kids are just gathering experience about these issues.

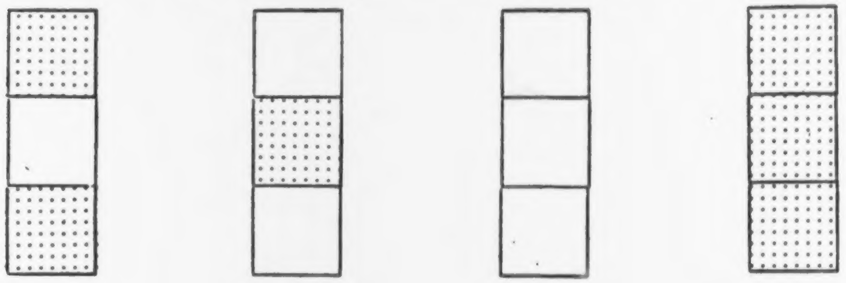
4. Looking for a law

The sequence of assignments from building one tower of a certain kind to several to as many as you can to every possible such tower makes a big jump from the first result made to the last. Every possible means that it is not just me, it is not just us who have not found more than 8 or 9 or 16 different towers of the given kind, but nobody ever will. Kids can come to this conclusion without overestimating their predicting power by discovering a structure. I would rather say by constructing a structure. This can be a stepwise process and can be achieved in various ways. A first attempt may be forming pairs of towers such as these:



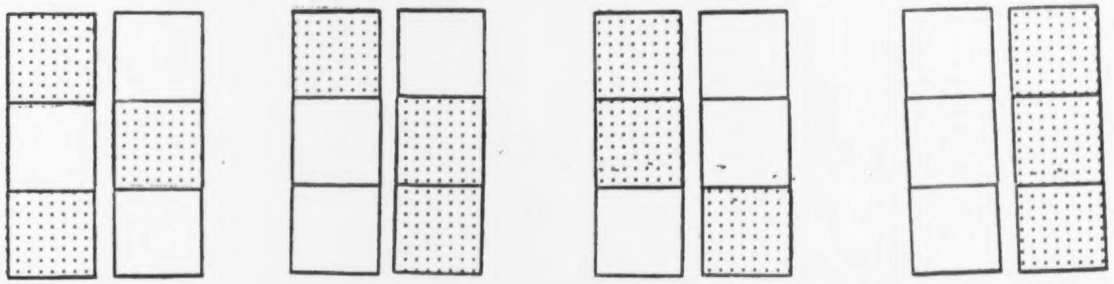


Others can be paired using the same principle:



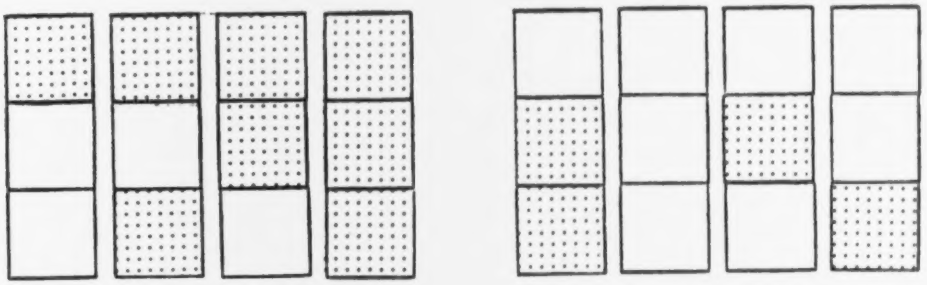
This structure may not be good enough to justify the number 8.

Another idea of pairing the towers is to form pairs (of the last four and also the previous ones, but in a different way):

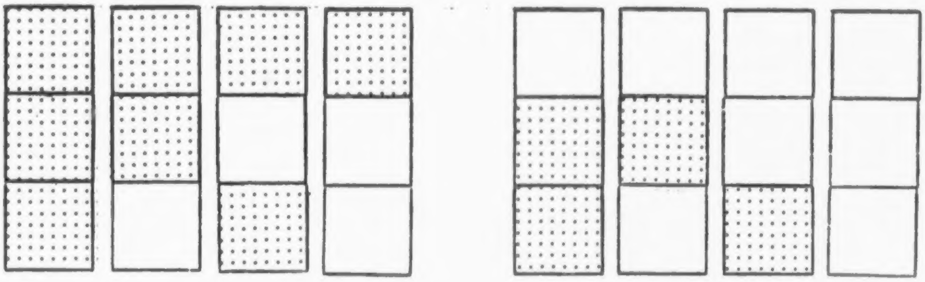


(Children of course, may mix these two structures with each other and make different ones.) The fact that each as exactly one pair corroborates the guess that no single one is missing -- but why should it not be that two are missing?

Still another approach to developing a structure is grouping together those with say, blue on the top and those with the other color, yellow on the top.



Further grouping can be based on the color of the block in the middle or the one on the bottom. Suggestions "can you make it nicer" are usually enough to improve the order up to the point when it becomes alphabetical, i.e. blue comes before yellow on the top, in the middle, and on the bottom, and groupings are as below.

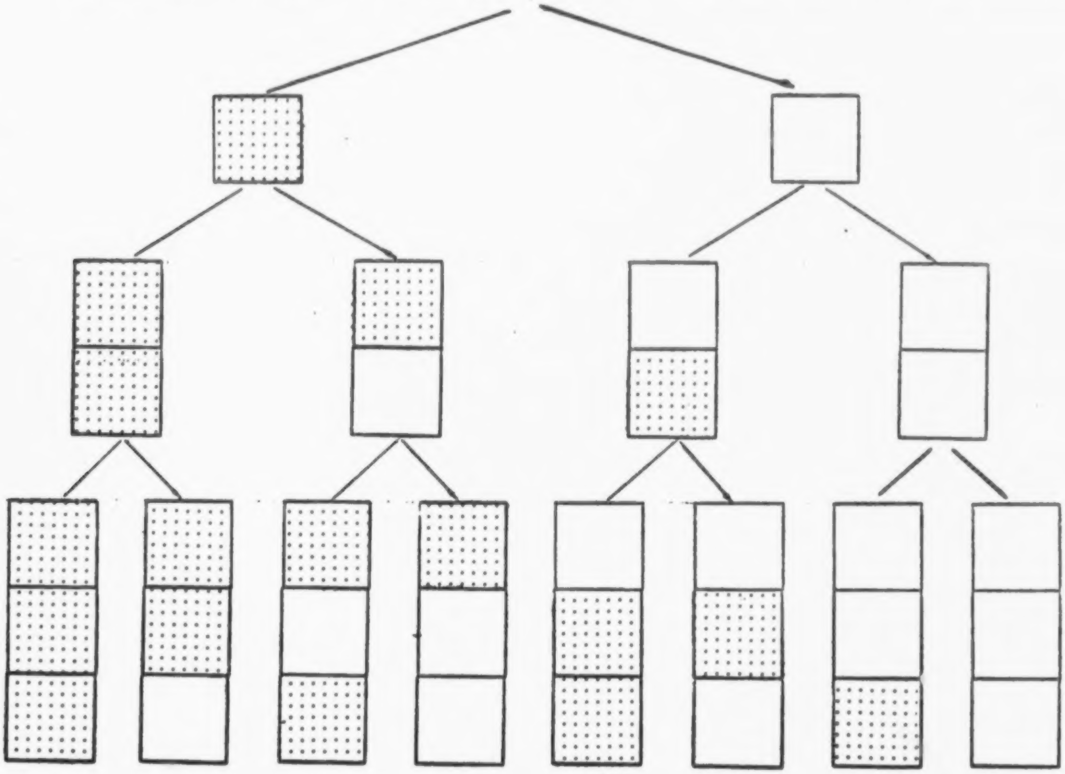


alphabetical order:

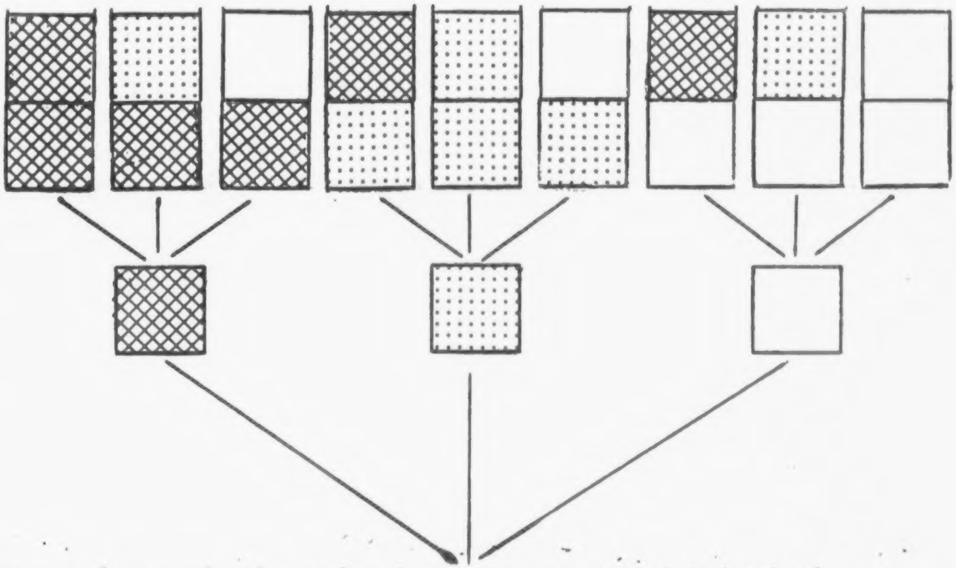
bbb bby byb byy ybb yby yyb yyy

For kids this is not a natural order. Looking up a name in a directory or a word in a dictionary remains a trial-and-error activity. The reason is clear: their math education did not include ordering sets of towers!

Tree diagrams are a graphical means of expressing the same structure.



This time building the more natural order would be for both the towers and for the tree to be built (to grow) upward. Using three colors and two stories the tree could look like this.



Symbolically, read upward, the order is again visibly alphabetical.

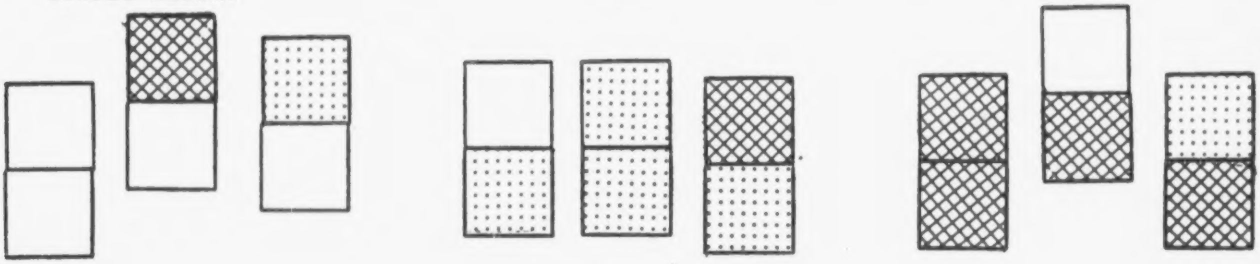
bb bg by gb gg gy yb yg yy

(We used an artificial means in getting the alphabetic order: Choosing the order of the colors: blue, green, yellow, to correspond to the alphabetic order of their initials: b, g, and y. In a wider sense the order of the letters in the alphabet is a casual one, and a row such as

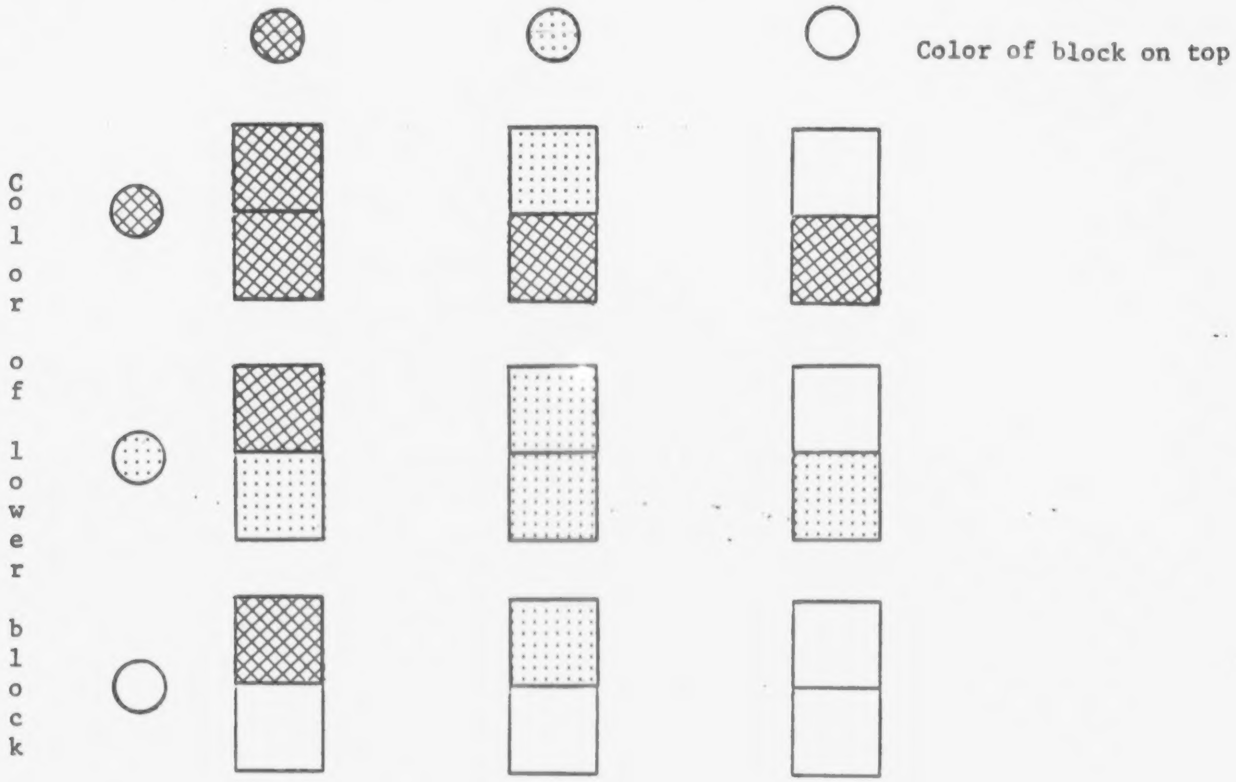
gg gb gy bg bb by yg yb yy

is not less alphabetical in its structure than the one above.)

Yet another structure, this time not an alphabetical one, would again start with initially towering unstructured sets determined by the color of, say, the lowest block.



Further structuring leads to forming rows and columns and rearranging the towers so that the dimensions "color of the lower block" and "color of the top block" have both their spatial equivalents.



A similar structure of the 8 tower example (two colors, three levels) can be realized in three dimensions: the towers are on the vertices of a cube. It is hard to go on with this kind of spatial representation to the fourth and further dimensions, but the tree representation lends itself flexibly to go on and on. The representation gives full account of the number of probabilities. The tree diagram is a proof, a non-verbal one, that there are exactly 8 (or 9, or 16, etc.) probabilities. The law is implicit in the tree structure.

Proofs are abstractions, and of a high level, even the intuitive (as opposed to formal) ones. Towers built of colored blocks, on the other hand, are very concrete, palpable, movable. Drawings and written symbols representing the towers, like those above, are somewhere between. There is an interesting point here: it is easier to reach the abstract level of a proof, if the starting point is as concrete as the building blocks. As movable objects, they lend themselves more easily to step-by-step structuration. Only for the mathematically mature person can a graphical (let alone a symbolic) representation be a good starting point. (S)he replaces the actual action by an interiorized one.

5. Varying the variable

Another way of coming close to combinatorial laws is looking for patterns in the numbers obtained, whether or not they are "proved" to be correct. The patterns will give the guideline to reassuring their being correct or revealing errors. In the example which is under discussion, we have two variables: the number of available colors and the number of stories, and this suggests that we use two-entry tables to tabulate the numbers of possible outcomes. So far we have these:

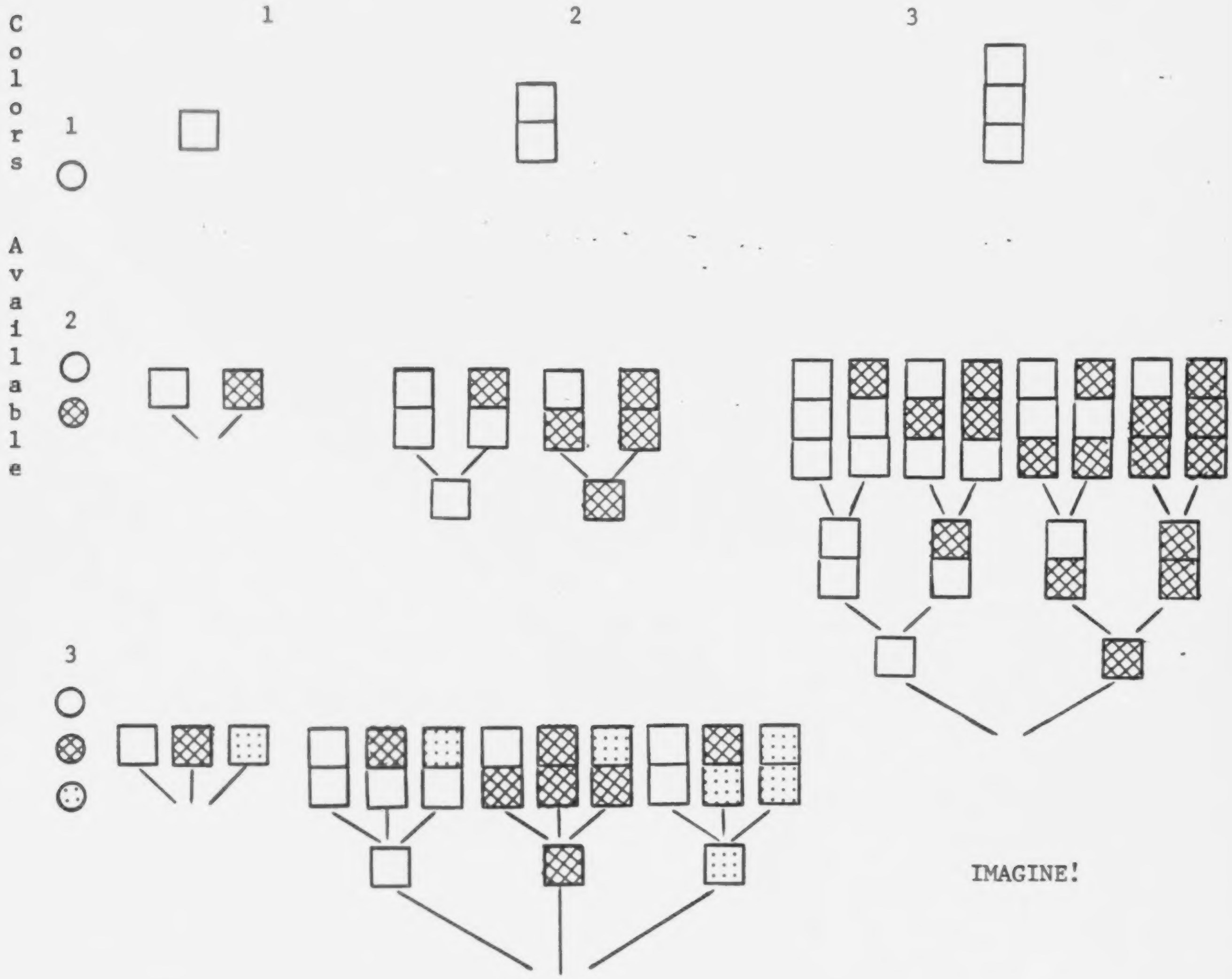
Heights of Towers (Number of Stories)

Number of Colors Available	1	2	3	4	5	6
1						
2			8	16		
3		9				
4		16				
5						
6						
7						
8						

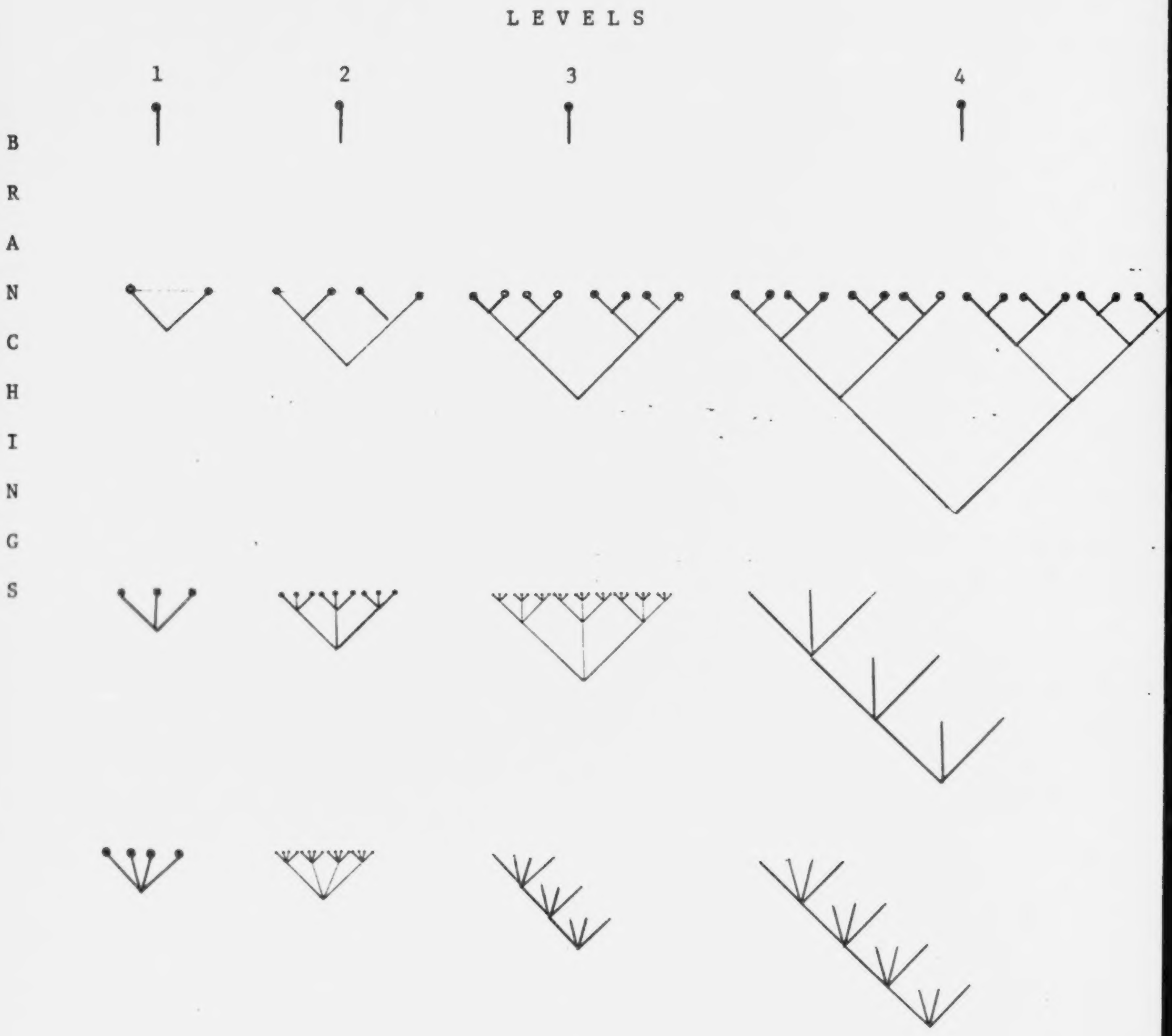
A combination of individual, small group and "frontal" (total class) activities can follow. The table with these numbers is hung or drawn on the chalkboard, visible for the class. The goal is to fill the empty spaces. Individuals or groups of two or three may decide which cells they want to take charge of; they write their initials into the cells and start working out the which go there. When they find a number, they write it in and choose another cell. Everybody is entitled -- or some may be charged with -- looking at the patterns in the table and putting in question marks to numbers they believe do not fit.

The two kinds of activities can be synthesized as suggested by this figure.

Heights of Towers (number of stories)



Another representation shows only trees and leaves the towers to the imagination (just as the missing parts of the trees).


LEVELS
1
2
3
4
BRANCHINGS

6. Changing the embodiment

In a mini-country license plates consist of three letters, no numbers, and the letters can only be A or B or both. How many different license plates can be given out in this country? Design all plates!

Now change all plates to allow A, B and C. If the license plates consist of two letters and no numbers, how many plates are possible?

Change the number of letters which can be used and the number of letters that can be put on a license plate and find the number of possible license plates in each case.

Children enjoy discovering the same mathematical structure in different embodiments. This enhances their abstracting power. They realize that the same sort of trees can be used to represent these situations.

7. Changing the rules

Students often understand the original assignment differently from how it was intended and this amounts to working with another rule. Let them do so, or let us come back to their rules later, or let us suggest to them to change the rule, or quite simply let us suggest different rules. Either way, plenty of things may happen:

a) The colors which are available should actually all appear on every tower.

b) Colors on a tower must be all different.

c) Adjacent colors must be different. (A different embodiment, designing flags which consist of two, three or more colored s stripes, may suggest this rule. The Australian flag is red-white-red. This is possible, but red-red-white or red-white-white would not make sense.)

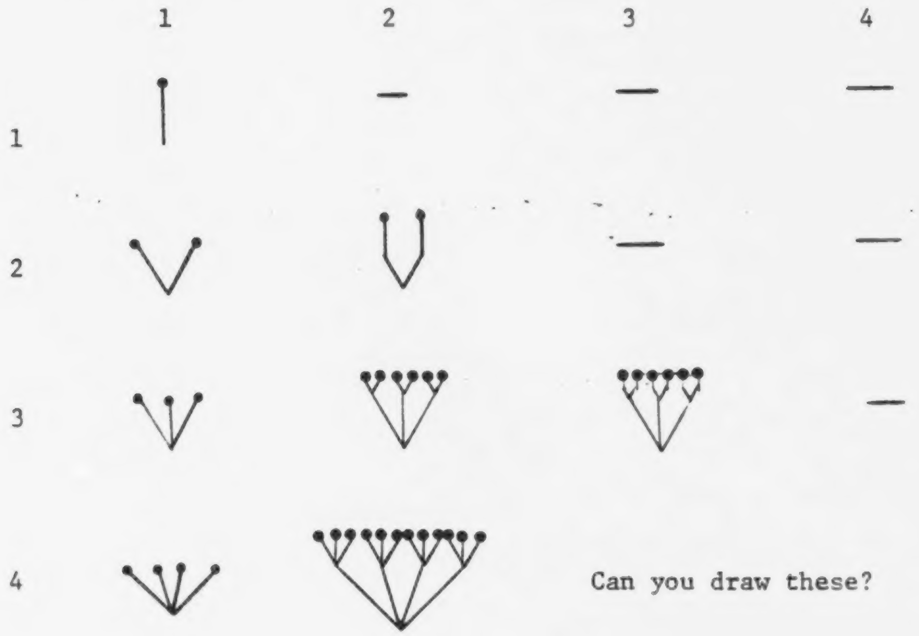
d) You must not have two towers with the same colors even if the order is different.

Take b) and work out the numbers in a 4 x 4 table. The result is shown below.

Heights of towers (length of 'words')

		1	2	3	4
Number of colors (letters) available	1	1	0	0	0
	2	2	2	0	0
	3	3	6	6	0
	4	4	12	24	24

The trees look like this



Can you draw these?

As the trees grow, at every higher level they push up less and less new branches - one branch less at each consecutive level. The reason is that the number of colors you can use is one less each time, since the colors used cannot be used again. (The actual construction of the towers conforming the new rule make this quite clear!) If you examine any row of trees, say row No. 3, you find that it can grow to be just one level with three branches: it can continue growing to be two levels, six branches; each of them still can push out one branch upward, and they will be six again; but then it is finished. This rule produces trees which do not grow up to the sky, while the earlier rule allowed trees to grow infinitely.

The previous rule, the one which let trees grow infinitely, can be described by the concise mathematical expression C^h where C is the number of colors available and h the height of towers. The present rule can be packed into the less widely known but useful formula $(C)_h$ which means take the whole number C, multiply it by the next smaller whole number, and so on, using h factors:

$$\underbrace{C(C-1)\ (C-2).\ .\ .}_{h \text{ factors}}$$

e.g. $(4)_3 = 4 \times 3 \times 2$

This is the number of permutations of four different objects, three of these each time.

8. Subtraction, probability, strategy-planning

Finding the operation you need (with what numbers? in what order?) is more important than performing the operation. Many people say this, but I think they are only partially correct. The argument usually goes this way: no calculator or computer can make you find out the operation, but they do help you to perform them -- actually they do perform them for you, more quickly and more accurately than you ever could.

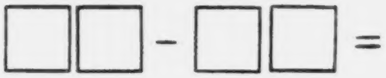
This is a half truth. First, because mental arithmetic with simple numbers (integers, decimals, even fractions) is and remains a powerful promoter of number awareness. Second, because with some practice most people are able to perform simple mental calculations more quickly and more accurately than by punching them into a machine (say, 0.8 x 0.5 or 1000 x 0.01 or 1000 - 993) and reading the result. Third, because mental arithmetic complemented (coupled) with skill in rounding and estimating is a protection against punch-in or read-off errors.

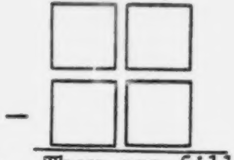
The conclusion (my personal conclusion) is that skill in multiplication tables, mental addition and subtraction of two-digit numbers and that sort of thing is to stay with us and belong to the elementary school mathematics of the twenty-first century.

So what is new about this? Conscientious and judicious teachers never neglected developing such skills. The novelty may be in the drills, not the skills. Paralyzed limbs may need specific exercises of specific muscles for rehabilitation, but healthy persons are better served by activities which engage a multitude of muscles, activities such as swimming or basketball. In other words, the answer is again integration. To be specific, here is a drill, different from the usual, to practice the subtraction of two-digit numbers and a lot else.

Each kid draws boxes for the digits the way they are used in writing subtractions.

This way or This way



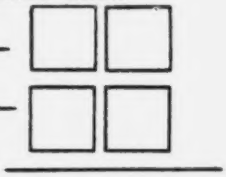


The goal is to make the difference as great as possible. They can fill the boxes in any order, but only with random numbers produced by rolling dice. After a number is given by the dice, children write it into one of the boxes which is still empty and cannot change it. After filling the third box they have no choice for the fourth random number. Those with the largest difference get three points, those with the second largest get two, those with the next get one and the others none. Then they start again.

Many nine-, ten-, and eleven-year olds develop efficient strategies to maximize their scores in the long run. They soon realize that

six at the first throw should go here———

and one at the first throw should go here—



but they are more hesitant about a five or a two. They realize that a six may follow the five and then chances are better for those who kept the upper left box free. Or a one (an ace) may follow the two, and then again they would regret not having left the lower left box open. Filling them with a six or a one, respectively, will never be regretted, not even if they are followed by another six or another one, respectively.

But can the players ever avoid being sorry in after-thought about the way they filled the boxes? A few rounds they actually play (and maybe a few more they play mentally) should convince the players that they cannot avoid that. In other words, there is no fail-safe strategy, though there are better and worse strategies. No matter where they write the first number, if it is a five or a two (or for that matter, a four and a three), in come cases they may regret that subsequently. The question is, how often and to what extent they have to regret their choices in the long run. This is where discussion with nine-, ten-, and eleven-year olds comes in (after playing the game a couple of times). Their intuitions lead them to different strategies which are then tested at further games. At a somewhat higher level, still in the elementary school, the strategies can be formulated more precisely, incorporated into computer programs and tested more precisely in a great number of trials. At a much higher level calculating the expected values related to specific strategies and finding the best strategy may be the subject of a research for undergraduates.

9. Estimation is involvement

In a second grade demonstration lesson in Hungary the teacher challenged a group of children to estimate the number of beads in five heaps with seven beads in each, the number of white Cuisenaire rods needed to cover seven yellow rods (of length of five centimeters), etc. The discussion after the lesson revealed that some teachers were reluctant to hear kids guessing that seven fives may be around forty-two or thirty. "Let them guess or estimate lengths, time intervals, weights, capactiy, whatever, but not the tables," they protested. "The multiplication table is something to know, not to guess."

To know when?

Traditionally, the multiplication tables had to be covered by the end of the second grade. This was followed in subsequent years by practice, practice, practice - at least five minutes a day. Recent suggestions encouraged the teachers to spread the time of learning the tables until the end of the third grade, leaving more space to meet the various cases in meaningful situations; to discover patterns in the tables and anchor their knowledge to the relationships they were finding; working out the products repeatedly, using concrete materials first, then only their heads.

Some teachers still advocate the "learning and practicing" sequence. (In some cases they order second graders departing for Christmas holidays to learn the tables at home, and come back to be rehearsed.) Yet an increasing number of teachers adopt the view that more time to learn and less drill to "fix" does not necessarily end in an inferior knowledge of the tables. But even the latter have an uneasy feeling about estimation as a starting point toward learning the tables.

Reasons must be strong, maybe not unrelated to sentences from "a child should not have an opinion" to "a child should have no will." If this is a correct conjecture, then the issue is a more general one about education, not necessarily school education or school math in particular.

My own view is that estimating, guessing, predicting, mentally representing, the future and expressing our opinion about it is a human ability which should play a greater role in education than it does now. All these activities (let me coin the word: guessivities) get kids personally involved in learning. The enormous difficulties encountered in the electronic conversion of spoken to written language are due to the fact that speech comprehension is more than decoding noises: it is a high level of human activity where guessing plays a major role. (The difficulties are almost the same in languages with nearly phonetical spelling.)

Focusing on math and science education, guessivity is a natural preamble before any calculation and any measurement. If kids take two beads out of a bag containing five beads, all identical except that two are red and three are blue, and find that doing this twenty times, they draw two beads of different color maybe ten times or maybe fifteen times, then they measure the probability and they find data like 1/2 or 3/4. Or they measure the expected number of bi-colored pairs in twenty draws and find the data 10 or 15. Alternately, they make assumptions about equally likely cases and they calculate the probability to be 0.6, the expected value to be 12. In any case, it should become a habit to start with guessivity: trying to predict the result. This gets them personally involved, and makes the rest more interesting for them. (Unless, of course, their only goal about learning is to get acceptable marks and pass exams.)

10. Estimations, just estimations

A calculation or measurement should be preceded by estimation (if only by a vague feeling about the result), but an estimation need not be followed by a calculation or measurement. Again I give an example about probability which I consider to be an important component of elementary school mathematics.

The question posed contains no hint to probability. It is simply about the number of minutes you need to arrive at school. Everybody writes his personal estimation: 12 minutes, 17 minutes, 35 minutes. . . (It should be made clear what you mean by arriving in time: when the bell rings, or 10 minutes before, as is usually required.)

The next question is: if you leave your house using the time you have written, are you certain you will not arrive late?

All of a sudden probability enters the scene. It usually does when you scratch the surface of a real life problem. In this case the discussion which follows reveals that several things may happen. An unexpected traffic jam. An empty gas tank. The car breaks down. You fall ill. An earthquake destroys the road. Some of these events are extremely unlikely. What does it mean, extremely unlikely? Try to express each of these by a number! 0.001? 0.0000001? (What do these numbers mean?) Anyway, they are not zero.

Next step: modify your estimation. How many minutes do you need (and consequently at what time do you have to leave) in order to be absolutely certain you will not be late?

Hours? Days? Whatever the safety margin, you can never reach absolute certainty. (You may die. Insurance companies have reliable estimations about the probability of that relative to age, health conditions, etc.)

If the probability of 1 of not being late cannot be achieved, then let us lower the level to 0.99. In other words, let us allow 0.01 probability for being late. Estimate the number of minutes you need under such a condition. You may be late once in twenty weeks. Does your estimation correspond to this level or do you want to modify it?

What if we lower the level to 0.1? Once in two weeks you may be late.

Being late half of the time isn't good, but how many minutes do you need if you set this as your goal?

How many minutes give you a chance of only 0.1 or 0.01 to arrive on time? (Once in two weeks, once in twenty weeks.)

How many minutes do not give you any chance?

Be careful and allow for such unlikely events as a helicopter picking you up when you leave and dropping you off at school. (Exclude Superman.)

Even so, 0 time certainty gives you no chance. That would mean infinite speed which is impossible. So probability 1 cannot be achieved, yet probability

0 can. After a few other estimated input and output values you will be ready to plot the function: number of minutes against the probability of arriving in time.

11. Afterword

Let me raise the question a third time: Is there any virtue in adding new topics to the Elementary School Mathematics Curriculum?

In the 1960's the answer was affirmative. Topics earlier considered too sophisticated for freshman or even sophomores - group theory, linear algebra, mathematical logic - let alone high school students, found their ways to the elementary school. Introduction to Finite Geometry by Kemeny, Snell and Thompson, published in 1957, was probably the first attempt in America to treat such topics in a freshman course and with success. It contains, among others, a chapter on linear algebra which introduces matrix multiplication through everyday applications. Less than ten years later Robert B. Davis' Explorations in Mathematics made available the same range of ideas to elementary school students (and their teachers, too!). I don't think these efforts were futile, even if we do not see many classrooms where the curriculum is considerably different from what we had thirty years ago. Beyond illusions and disillusions, we now see more clearly both the long term goals and the ways toward them. Yes, new topics can be added to the curriculum, amalgamating them with precious old material, keeping the continuity and striving for progress at the same time. The world is not the same as it was thirty years ago, there is no way back -- only forward.

MATHEMATICS FOR THE TWENTY FIRST CENTURY

A. I. Weinzweig

There have been a great many changes in the mathematics curriculum over the last several decades, affecting not only what mathematics has been taught, but how it has been taught and where the emphasis has been placed. First the "New Math" swept across the country like a tidal wave, and now we are being engulfed by the "Back to Basics" movement, with the "Behavioral Objectives" and several other movements in between. Each left its imprint on the curriculum. These shifts from one extreme to the other were enough to leave one's head spinning! One might well ask what will be the direction and nature of the next shift! Looking ahead, what will the Mathematics Curriculum be like in the Twenty-first Century? This might seem, as a first reaction, like looking far ahead, well into the future, calling for some long-range projections. However, when one realizes that the children in schools today will be functioning for most of their adult lives in the Twenty-first Century, it becomes clear that we must deal, not with the future, but the immediate present. In the light of this fact it may be more relevant to raise some other questions. How well are we preparing our children for adulthood in the next Century? Are they getting an adequate foundation to meet their future needs? It will be necessary, in order to answer these critical questions, to get some idea of the nature of this new world so that we may better understand the knowledge and skills that our children will need to function effectively in it. This will put us in a better position to judge present practice in the schools.

It is dangerous to predict what will be. Nevertheless, it is possible to extrapolate from the present to the future by examining the changes that have occurred as we progressed from the past to the present. From our perspective, looking back, one of the most significant factors affecting this change has been the widespread use of machines. These have greatly expanded our physical power. Where before it would take twenty or thirty men several days to dig the foundation of a building, today one man on a bulldozer does the job in one day. This new power created a great deal of anxiety and concern, a fear that machines would displace men in the labour force giving rise to large scale unemployment. There were, of course, dislocations in society -- some of them quite severe. The net result, however was quite different. This extension of our physical power through the use of machines opened new horizons. It created the need for, and made possible, the undertaking of many more projects on a larger scale. Where before twenty or thirty men wielding picks and shovels were needed to dig the foundation for one building, now twenty or thirty men operating bulldozers are needed to dig the foundations for many more and larger buildings! Thus, instead of eliminating jobs, machines have given rise to a new situation where more, not fewer men are needed. Nevertheless, there has been a shift in the nature of the labour force. In the past, the main prerequisites for the job were brawn and physical stamina; today, it is the skill to operate a machine -- the bulldozer. The NUMBER of men required has not decreased but the level of skill required of these men has gone up considerably. Thus the needs of society are not for fewer men, but for much more highly trained individuals! This has led to other demands leading to new industries and the concomitant need for more labour.

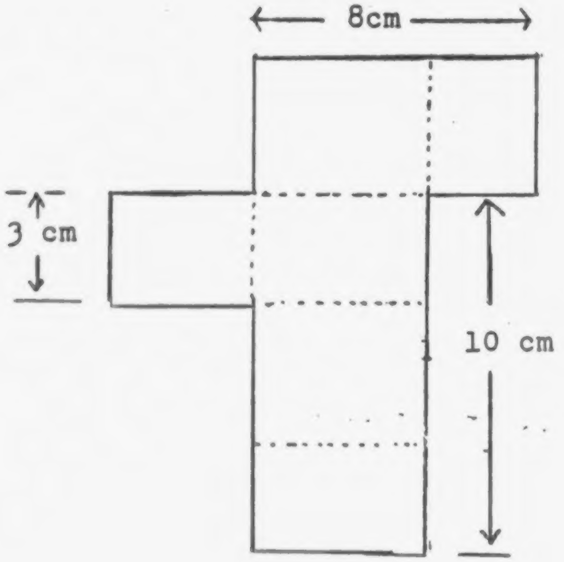
Today we are witnessing the beginning of a new revolution, as profound and as far-reaching as the one produced by machines. Where the old machines greatly extended our physical power, the new machines are greatly extending our mental power. A project which would normally take a number of people a great effort for a considerable time can now be done by these new machines -- computers -- in a negligible amount of time. Calculations which required teams of people can now be performed by one person on a computer in far less time and with far less effort. Routine tasks, in banks and offices of various kinds are now being done by computers, drastically reducing the need for "white collar" workers. Similarly, "blue collar" workers performing mechanical operations on assembly lines will be replaced by computer robots. As with the old machines, this will result in the initial displacement of many people as their jobs are taken over by computers. However, once again, the extension of our mental power through the use of computers makes possible the undertaking of many new tasks hitherto far beyond our capabilities. Thus, the needs of society will change, requiring not fewer people, but people with greater and more sophisticated skills! Just as now there is little market for unskilled labour, so in the future there will be little need for people without these more sophisticated skills. These changes are taking place at a very rapid rate NOW. The future will only accelerate this pace of change!

As we advance into the "Computer Age", one thing is clear, the skills which will be most valuable are not those which these machines can perform more effectively, but the skills required to perform more effectively on these machines! This requires a level of understanding beyond the ability to use rules without knowing why they work. A much deeper insight, a level of understanding of the processes and concepts of mathematics so as to be able to apply them in diverse situations, is essential. There is a vast difference between the abilitity to multiply

3 x 4 x 5

correctly and the ability to recognize that if the piece of tin with dimensions shown in Figure 1, is folded along the dotted lines, the volume of the resultant box will be 3cm x 4cm x 5 cm. The former requires only what we call algorithmic understanding while the latter a higher level or analytic understanding!

Figure 1.



Similarly, only algorithmic understanding is needed to calculate the area of the rectangle in Figure 2, but to determine the area of the regions in Figure 3 takes analytic understanding! It is this deeper, analytic understanding of mathematical concepts that must be cultivated.

Figure 2.

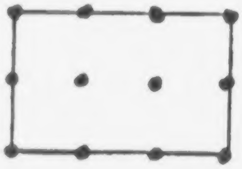
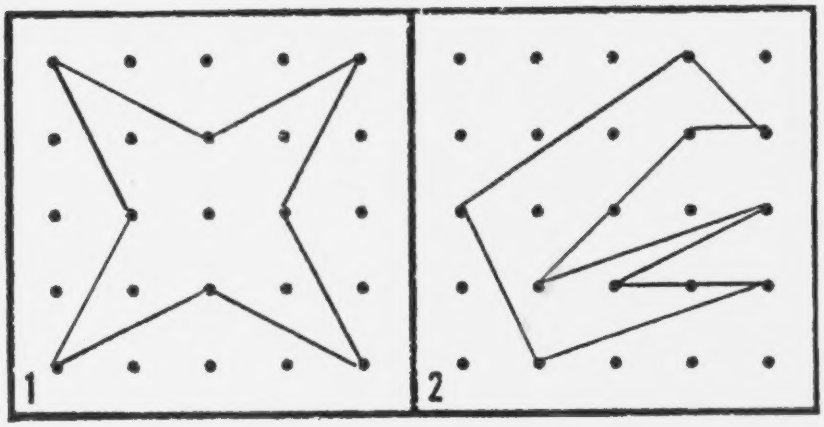


Figure 3.



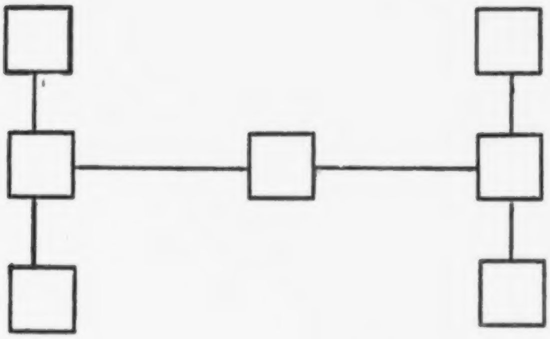
How do present practices measure up to these requirements? Not too well, I am afraid. The primary emphasis in schools today, with regard to mathematics, is on developing skill and proficiency in computation and on algorithmic proficiency almost exclusively. This, no doubt, is the result of the "Back to Basics" reaction to the excesses of the "New Math", to which many people attributed a general reduction in computational abilities. Regardless of the validity of this allegation, the ability to compute rapidly and accurately, to which so much time is devoted in the mathematics programs, will not be, is not now, and has not been for some time, an important skill. Indeed, this is an excellent example of focussing on a skill which a computer (and even the simpler calculators) can do far more effectively, for such routine calculations have for many years already been done with machines. Are we then suggesting that children should not be taught to calculate? Sacrilege!! Heresy!! What if the battery runs out? Of course we must still teach children to calculate or compute just as we must still teach children to write, even though we have typewriters! However, penmanship is no longer emphasized as it once was, since most important letters or other manuscripts are today written on a typewriter (and increasingly on a word processor using a computer such as the one which I prepare this manuscript). Similarly, it is neither necessary nor desirable to devote so much time and effort to developing the high degree of competency in calculating to which we now seem so committed. The question of what happens when the battery of the calculator dies is of as much (or as little) concern to me as the question of what happens when the ink in the ballpoint pen runs out! (It has been some time since we have heard this argument against the use of ballpoint pens in schools!) Clearly, you replace it! It is not that there is anything inherently wrong with proficiency in calculation, it is just that this is a low level ability and the limited time available for school mathematics would be spent more profitably if the development of higher level abilities and understandings were emphasized.

A similar view is now shared by many others. Indeed, "We must teach Problem Solving!" is the latest salvo in the ongoing battle to improve the teaching of mathematics. "Problem Solving" has now joined "addition", "place value", "fractions", "word problems", etc. as a topic in the curriculum. However, the conscientious teacher who seeks to follow through on this call to action soon runs into difficulties. What is "problem solving"? How does one teach it? What in fact is a "problem"? How widely this varies from one textbook to another, and from person to person is evident from the following examples:

a) $3 + 4 = ?$

b) John has 3 dollars. His aunt gave him 4 more dollars. How much money does he now have?

c) Arrange the digits 1, 2, 3, 4, 5, 6, 7 in the boxes below so that the sums of the digits on the horizontal line and on each of the two vertical lines are all the same.



Clearly, from some grade level on, we would expect (and hope) that (a) and (b) would pose no problem for most students.

The use, above, of the phrase "no problem" was deliberate, to call attention to the different meanings often associated with "problem". In many classrooms it is used to apply to an exercise, in the textbook or elsewhere -- "Do problems 1-10." In other cases it relates to difficulty -- "He had a problem with his boss." There is even some ambiguity in "difficulty", for while most of us would find the following difficult,

(d) 343,782,489,374 x 29,863,589,682 = ?.

some of us would also find difficult the problem of restoring the Rubik's cube to its normal state with each face one color. We are, quite obviously using "difficult" with two different meanings, for no doubt most of us would find (d) tedious but achievable whereas many of us would find the Rubik cube problem frustratingly unachievable! In much the same way, for certain quite young children question (a) would be incomprehensible whereas question (b), for these same children, would be a "real" problem, difficult but solvable.

Any discussion of problems or problem solving, to be meaningful must make perfectly clear how these words are used. (While this comment may seem rather trivial and obvious - as it should be - it is amazing how much and how often confusion is caused by the failure to recognize this "obvious" fact!)

We therefore present the following working definition:

> A task is a problem for an individual if that individual does not have the requisite knowledge immediately to carry out the task. He must in fact, create the necessary knowledge either by synthesizing knowledge he does possess or by inventing new techniques. What is a problem depends not only on the task but also on the background, experience, and knowledge of the individual confronting the task. It thus denotes a relationship between a task and a person and is therefore relative.

If someone knows a sufficient set of moves and a strategy for applying them then the task of restoring the Rubik cube is not a problem for that person! Indeed, any task which can be carried out by means of some algorithm - a sequence of steps or rules - is not a problem in this sense.

It is clear from the above specification of a problem that problem solving requires an analytic understanding of the knowledge involved. Problem solving can thus be an important element in fostering the kind of analytic understanding of mathematical concepts which we deem so essential. However, it should not be merely one of several topics, but must pervade the entire mathematics program. Indeed it should be the foundation on which the whole curriculum is built! There is, we believe, a symbiotic relationship between problem solving and concept development which is fundamental to both, and to clarify this important point, we must look more closely into the nature of mathematical concepts.

Mathematical concepts arise out of a specific context in response to a particular need. In order to deal more effectively with a recurring problem, we invent a concept. Quite frequently this problem situation involves some question of the form "What would happen if. . .?", where the outcome of a certain action must be determined. We could, of course, carry out this action but that can at time be impractical or inconvenient. However, using the concept we can often predict the outcome of this action WITHOUT CARRYING IT OUT! This is what gives power to the concept and makes it so useful as illustrated by the following example.

> I would like to purchase a certain dresser in a store for a special place in my bedroom. Will it fit? One way of finding out would be to bring the dresser home from the store and try to fit it into the space. An easier solution would be to compare the length of the dresser to the length of the space in the bedroom.

In effect then, the concepts of length and measurement enable us to determine the result of actually trying to fit the dresser into the space, without really having to do so. In this way we can predict the outcome of the action without carrying it out, which is a great deal simpler!

Many of the concepts of mathematics arise in a similar way. Thus not only is mathematics important in problem solving, but problem solving is important in the creation of mathematics. Returning to our example, not only is the mathematics used to solve the problem, but it is problems such as this that gave rise to the concept used in the first place!

That there is more to the story will be evident from the following consideration of fractions. Everyone knows that 1/2, 2/4, 3/6 and 4/8 are all equivalent (or should I say equal?) fractions. Without worrying about the distinction between "equivalent" and "equal" what does this mean? Surely, at a minimum, it should mean that any statement involving a fraction should remain as true and meaningful when the fraction is replaced by an equivalent (equal?) fraction as it was before the replacement. In that case the statement

> The numerator of 1/2 is odd.

which is true, should remain so when the fraction is replaced by 2/4, which of course, is not the case. The usual explanation for this is quite simple. The statement is not about the NUMBER 1/2 but about the NUMERAL "1/2" where the quotation marks indicate that it is the name or numeral that is being referred to and not the object being named, the number. It makes sense to talk about the numerator of the numeral "1/2" but not of the number 1/2. If then "1/2" and "2/4" are two numerals or names for the number 1/2, should we not write

$$1/2 = 2/4$$

for is that not the meaning of the "=" sign in mathematics? Why then do we just as often say they are equivalent and in many of the elementary mathematics textbooks find lists or sets of equivalent fractions?

What is the number 1/2? The stock answer is that it is a rational number, and a rational number is usually defined as follows:

> A rational number is any number which can be represented in the form a/b where a and b are integers. (It is customary to include the condition that $b \neq 0$, but since in fact for any a, a/0 represents no number, this condition is redundant!)

This in no way tells us what a rational number IS but only WHICH numbers are rational. It serves merely to distinguish rational numbers from non-rational or irrational numbers among the real numbers. It certainly offers no help in understanding what is a rational number! Leaving aside for the moment this vexing question, consider the following situation. There are eight children, four of whom are girls. It would make sense to say that 1/2 of the children are girls, or that 4/8 of the children are girls or even that 2/4 of the children are girls, but no one would ever say that 3/6 of the children are girls or 6/12 of the children are girls. In spite of the fact that 1/2, 2/4, 3/6, 4/8, and 6/12 are all equal (or equivalent) fractions, in this context replacing "1/2" by "3/6" would not be meaningful! There seems to be some disparity here between the mathematics, which says that "1/2" and "3/6" are interchangeable when we are talking of the number as we are in this case, and normal usage. Indeed it is clear from the above discussion that there is a great deal of confusion with the whole subject of fractions or rational numbers. (Is there any wonder that children have so much trouble with this topic?) What is not so clear is that this same confusion exists with regard to most concepts in mathematics. When we ourselves can easily deal with them, we are not so conscious of it. Unfortunately, the children are usually acutely aware of this situation!

The process of concept formation is a dynamic one. A specific problem situation in a particular context gives rise to a certain conept. As new, slightly different sitatuions are confronted, the concept is extended to apply to them as well. In this way the range of applicability of the concept is enlarged, that is, it is generalized. However some modifications may be necessary! In this way we generalize the concept. At some point, someone observes that several concepts, each arising from a different context, can be regarded as different manifestations of a new concept. All these different concepts can be subsumed under a single concept, one level "up" from the contexts that gave rise to the original ones. The scope of this new concept is thus further increased, but it is now one level removed from the initial contexts. It is more abstract. This process of abstraction continues. At each level of abstraction the concept becomes more general, applying to a much broader range of contexts, but further removed from any one of them. As we "rise" above the contexts in successive levels of abstraction we gain a broader view, but lose some of the detail as we get further from direct contact with any particular context. This process of abstraction and genralization leads eventually to an abstract, context-free formulation of a general concept. Our notion of rational number is such a general context-free formulation. This formulation, free from any particular context is very powerful, for it enables the application of these concepts to new situations.

Moreover, it permits the shifting of a problem from one, perhaps less familiar context to another more familiar one, where it can more easily be solved. The solution can then be shifted back to the original context. This is related to the process of modeling. One of the difficulties, however is that we must learn how to apply the general concept to a particular situation, where it may be more restricted. How does this manifest itself in practice? Consider again the concept of fraction as it appeared in the discussion above. There are a number of different concepts of fraction, each arising from a different context subsumed under the general concept of rational number. In the situation encountered above, we can express the fact that the eight children can be considered individually and four of them are girls, by the assertion that 4/8 of the children are girls. We could also group the children into pairs so that two of the pairs are made up of girls and describe this by saying that 2/4 of the children are girls. Finally, we can express the fact that there are as many girls as boys by saying that 1/2 of the children are girls. In each case, the information that is being communicated is that children can be separated into groups each with the same number of children, each made up only of girls or only of boys. The denominator tells us how many such groups there are and the numerator tells how many of them are made up only of girls. Notice that there is a great deal of information that is missing. We have no idea how old the children are, what they are wearing, what the weather is like, etc., etc. If we were simply given the information that 2/4 of the children were girls, we would not even know exactly how many children there were. We would, nevertheless, know that there could not be 7 or 5 or even 2 children, for in each of these cases we could not separate the children into 4 groups with the same number of children in each group, that is the number of children is not divisible by 4. On the abstract context-free level 2/4 and 4/8 are equal, but in some particular context they may not say the same thing. What then is the use of the general context-free concept? It tells us that 1/2, 2/4 and 4/8 are equivalent and since 2 and 4 are factors of 8, if we could describe a situation as 4/8 of the children are girls, we could also say that 1/2 or 2/4 of the children are girls. In other words, while on the abstract level two fractions may be equivalent, whether they can in fact be interchanged depends on the particular context. It is therefore important to recognize, in developing a new concept, that in fact there are different concepts in different contexts so that we are really dealing, not with one, but several concepts.

This brings up another important point which is often overlooked. Fractions are used above to describe a situation. Indeed, a critical aspect of most problem situations is the need to communicate - to describe both the problem situation and the solution. This necessitates the development of languages, spoken languages - terminology - and written language - notation. The failure to deal appropriately with the development of the terminology and even more so with the notation is the cause of many of the difficulties students encounter in mathematics. Once again, an example will help to clarify this point.

One of the most frequent errors made by children when confronted with

$$4 + \_\_\_ = 11$$

is to place "15" in the box. While it is certainly the wrong answer to the intended question, it IS the correct answer to a different question, namely:

$$4 + 11 = \_\_\_.$$

This would suggest rather strongly that the incorrect response indicates, not an inability to perform the task, but rather a failure to comprehend just what the task is. In this case, the error is not the result of a lack of skill in or understanding of the operation involved, but more likely a failure to understand the notation, which is a language difficulty!

No one will disagree with the assertion that it is important to develop, in children, a deeper analytic understanding of concepts. Indeed, many will argue that this is what they are doing, or at least trying to do! We speak rather glibly of teaching the child the concept of, (say) a fraction or rational number, or of helping the child learn this concept, as if the concept exists outside of the child and our task is somehow to implant it in the child. This view is implicit in most of what goes on in schools and even in those textbooks which argue for more child involvement and advocate a laboratory, activity oriented approach, when they speak of embodiments, or concrete models for the concepts involved. Our view is quite different, for we do not believe that concepts exist external to the child, but they must be constructed, internally, by each individual child. Each one must reinvent the wheel, if the wheel is to have meaning for him or her. However, the child invents or discovers the wheel within the context of a society that has already invented the wheel, where the use of wheels abound!

The interests of any society require that knowledge be passed on from one generation to the next. Schools are the main vehicles for this in many societies, including our own. However, if we attempt to teach children by telling them what they need to know, then at best they will achieve an algorithmic understanding - the ability to apply their knowledge only in a routine manner in familiar situations. This is, in fact, the most common approach in schools today with results that are all too familiar and unsatisfactory.

> Recently, a visitor to a fourth grade class was informed that the children were quite expert in multiplication. Each time he gave them two numbers, they multiplied them correctly and rapidly. When he asked, "In the last school I visited, there was a room with 12 rows of chairs, each with 15 chairs, how many seats where there?" There was a long pause until finally one child asked him, "Do we plus or times?"

Unfortunately, this was not an isolated event, as every teacher knows!

This still leaves unanswered the central question. How DO you teach children so that they develop an analytic understanding of concepts? The answer is very simple - you don't! I do not believe that anyone, even the most gifted of teachers, can teach anyone mathematics. It can only be learned. Each child must develop and construct knowledge for him or her self! What then is the role of the teacher in this enterprise? He or she must facilitate this activity of the child, not hamper or hinder it. How is this to be done? We do not have all the answers, but we do have some suggestions.

Given a concept that we want to "teach", the first step is to analyze it, not from the perspective of the mathematician, but from the perspective of the child. We must consider the different contexts from which it arises and recognize that these context-specific concepts differ from one context to the next, and from the usual context-free formulation. With respect to fractions, in the context presented above, fractions are introduced as a way of describing a particular situation. The statement

1/2 of the children are girls

says less than the statement

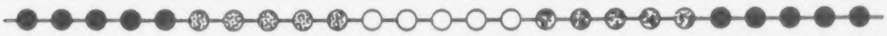
2/4 of the children are girls,

for from the first statement we can know only, for example that there are an even number of children, but the second case tells us even more; that the number of children is a multiple of 4! However, both tell us that there are the same number of boys as girls, and if this is the pertinent information, then 1/2 is equivalent to 2/4 in that both give this pertinent information. Similarly, one half dollar and two quarters are different, but equivalent in value. On the other hand, the point on the number line with coordinate, 1/2, is the same as the point with coordinate, 2/4. In this case, the two fractions are labels or names for the same point and hence are equal. We are however, dealing with two different concepts arising from different contexts, both of which are subsumed under the general context-free concept we call a rational number.

The teacher must create for the child FOCUSSED experiences, usually problem situations where the child can come up with a solution. These should be so structured as to direct the child's attention to looking for an "easier" solution through the invention or discovery of a particular concept. If a picture is worth a thousand words, then an illustration should be worth at least a couple of pages of text! Some five year olds were given a collection of solid models and asked to separate them into two piles so that any two models in one pile were the same in some way and two models in separate files were different. They were asked to explain how they were the same. This activity focussed their attention on some of the properties of the models; some of them rolled, some did not: some rolled when placed in one position but not in another; some were "flat", that is, had parallel faces, others had "points", like the pyramid and the cone; they counted faces, edges and vertices. They had a problem keeping track of which faces (or edges or vertices) they had counted until one of the children got the idea of sticking a bit of modeling clay to each face as they counted it. Later, they were given some larger blocks with rectangular faces, and construction paper rectangles which they were to use to cover the block. The task was to match the rectangles with the corresponding face. One of the children observed that each of the rectangles "fit" on each face, that is the corners matched (the right angle!), but not all over, others fit along with side (edge) but not all over. At one point they were asked to bring a box of blocks which was across the room. It was too heavy to lift and there was not enough room around the box to accommodate enough children to lift it. They could not slide it as the floor was covered with a rug. This was shortly after they had worked with the models and when a cylinder "accidentally" rolled off the table, the attention of one of the children was attracted to it and he stared at it intently. He then picked up the cylinder and with the help of some of the other children, managed to tilt the box enough to slide the cylinder under it. They were then able to roll the box. "It's like a wagon!" exclaimed one. They then tried other models and discovered that with the cylinder they could only go in one direction. The cone only allowed them to go in circles. The main point of these tasks was that they focussed the children's attention on certain aspects of the situation and that they involved problem solving. The children discovered many geometric properties, but in a context which made them aware of how these could be used. They were not simply told, but began to formulate these concepts themselves. The concept of right angle, say, that they had begun to construct was not the usual context-free notion encountered in Geometry (with a capital "G"!), but one which arose out of a particular context and was very useful in that context!

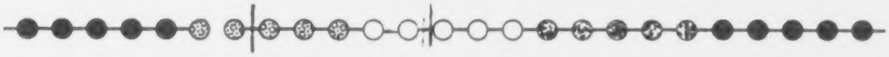
We present one last example to illustrate two points. Addition is a concept which enables one to state the result of counting without doing the counting. Given two piles of books on a desk, most people if asked how many books there were, would count. If, however, Jane came in with a pile of books and placed them on her desk with the announcement that there were 9 books and then Bill placed the second pile beside it saying that there were 8, then it would be more efficient to add! In the first case, when the number of books in each pile is not known, then it is necessary to count 17 books. There is little advantage to counting the number of books in each pile separately and then adding, and even some disadvantage. However, when the number of books in each pile is KNOWN, then addition gives us what would be the result of counting (which is how we would answer the question of how many. . .), without counting. The great focus in our schools and textbooks is on counting as a way of developing the concept of addition, which is treated as something which exists. It is generally regarded as context free as is the usual notation which is really quite abstract! An alternative approach provides a context from which arises a particular concept of addition. Twenty-five beads are strung on a stiff wire, the first ten are red, the second ten yellow, and the remaining five red. The first five beads of each color are one shade and the second five another. (Figure 4.) Children can "see" up to five beads, that is they know how many there are without counting. They can easily "see" two or three more than five or less than ten or more than ten. Initially all the beads are to the right and the child is asked to count out, that is shift to the left a given number of beads. He is given a great deal of practice in this task with the numbers 5 and 10 occurring quite frequently. It does not take too long for him to recognize that 5 is all the beads of the first shade of red, and 10 is all the red beads, and he shifts these without counting. He soon learns to shift the beads without counting for other numbers. He is also noticing other important facts because of the way the activity focusses his attention. Thus, when 7 beads, say, are "counted" out he sees this as 3 more to get 10 or 2 more than 5.

Figure 4.



Once the structure of the bead frame has been discovered, we are now ready for the next task. The child is asked to first count out 7 beads and then another five and asked how many have been counted out altogether. From the previous activities, the child has already "seen" that 5 beads can be separated into 3 beads and 2 beads, that 2 is the complement of 3 in 5 and that 3 is the complement of 7 in 10, so that 5 more than 7 will be 2 beads into the yellow. In a short time, the child would shift first 7 beads, then 5 more and give the answer, 12, without counting! The child begins to "visualize" number facts IN THIS CONTEXT! After this activity comes the problem of recording what we have done. This involves developing a notation to describe what has been done in this context. When the child has first shifted 7 beads and then 5 beads he has shifted 12 beads. At that point, certain information is lost, since the 7 beads are no longer separated from the 5 beads. We could use a paper clip to separate the 7 beads from the next 5 beads. Subsequently, this information can be recorded on a PICTURE of the bead frame by placing a stroke after the 7 beads and a double stroke after the 5 beads. (Figure 5.) A simpler notation, but more abstract and further removed from the context, but nevertheless suggested by it is the following:

Figure 5.



7 , 5 ⟶ 12

where the comma separates the 7 first counted from the 5 and the arrow indicates that this shifts over 12 beads. The notation is introduced slowly and to a very large extent, nonverbally. The child is not TOLD what the notation means but it is used to record the action as it takes place. Once the notation is understood it can be used to communicate questions. Thus, if the teacher writes on the board, without saying a word,

6 , 5 ⟶

and waits, one of the children will volunteer the answer, 11, which the teacher then writes in the appropriate place. This builds up the language and leads easily to such questions as

7 , ■ ⟶ 12

where the children were told that some spilled coffee had obliterated one of the numbers and the children were asked to help reconstruct the situation. One child immediately volunteered 5 and when asked how he knew, explained "3 and 2". On being asked to explain further, he took the bead frame and shifted over 7 beads without counting, saying "seven!" He then grasped the second yellow bead and shifted it slightly to the left, shifting 5 more saying "and this is 12." Then pointing to the 3 red and 2 yellow, he continued "3 and 2, 5."

The structure of the bead frame leads children to visualize number facts and arrive at the answer without counting. This obviates the need of the child to count on his fingers. The "addition" here derived is context bound as is the notation. However, because of this, the child has little difficulty since he can make use of the context in solving a problem. Of course, further activities with other contexts must be provided and experiences which make the child aware that results in one context are transferable to another context. This provides an advantage to the development of a context free concept and notation. However, the child is free to revert to any context to arrive at an answer!

The approach indicated in the preceding illustrations can be applied to any concept in mathematics. It is not simply a pedagogiqal technique, but rather reflects a particular view of the nature of mathematical concepts, of children and of the way children learn. We think it is a valid one which has been borne out by our own experiences. Moreover we find that when children are involved in this way as active participants in the learning process, they develop much more understanding. After all, is not that what will be important for the Twenty-first Century?

THE TEACHER AND CURRICULAR CHANGE

Fred D. Carver

The classroom teacher is in both the best and worst position to effect curricular change.[1] From the simplest perspective, the teacher is the point in the educational process where meaningful modifications in content and method occur. That is to say the teacher is the first and last rendezvous in the school's protracted engagement with pupils. And, regardless of guidelines, commercial materials, and other assistance available, day-to-day instructional activities are determined by the teacher. In specific instances, the teacher who has a definite idea about what content to teach, which materials to use, or how to structure the instructional activity can most often implement the idea.

Simplicity evaporates quickly in reality, however. While teaching is a lonely, isolated job, most teachers come under the influence of colleagues even if they do not share instruction for a group of pupils; unwritten rules about the limits of autonomy develop quickly, are informally enforced, and persist for long periods of time among teachers. School district curriculum guides and commercially prepared materials constrict teacher choice of content, materials and method to some degree. Administrators and supervisors, through district monitoring of school/district curriculum plans or by more general policies and norms, define parameters for individual teacher autonomy to change the curriculum. A myriad of external forces, from legal determinations to parental attitudes and expectations, serve to limit teacher choice in conducting classroom curricular activities. So what appears to be a simple matter of the teacher being able to effect curricular modifications is, in reality, a complex issue indeed.

Changes in the role of the teacher in curricular change have paralleled the development, and attendant bureaucratization,[2] of the system for public schooling in this country. When the dominant pattern of public schools was the one-room schoolhouse, the teacher determined the curriculum, restricted only by the availability of books and other materials. Administrators and supervisors, acting for boards of education, assumed responsibility for devising an articulated curriculum throughout the grades as the single schools grew into school systems. Collectively teachers were (are) most often involved in the conceptualization of and decision making about curriculum content, materials and general methods. This impacted on the individual teacher's responsibility for making major curricular decisions.

The public schools' and teachers', most prominent experience with broad curricular change follwed the launching of Sputnik in 1957. A strategy for major reform, especially of secondary school curriculum, combined non-public school (subject matter) experts and federal financial support. Curricular materials, explicitly prescribing content and method, were prepared for teachers to follow. This strategy, perhaps inadvertently, appeared to place the teacher in the role of passive carrier of curricular change. For a variety of reasons the curriculum project movement was significantly less effective than any of its advocates envisioned: misconceptions about how to effect changed based on faulty assumptions (at best) about the culture of the school, pre- and in-service education programs not sufficiently connected to the new curriculum, and an even less knowledgeable and understanding public--to name a few.

Implicitly the curriculum project movement focused attention on improving public education by concentrating on individual teachers and classrooms, in the special case, through improving the curriculum content, materials and methods. Within the last ten years, the prevailing view has come to be that effective change efforts, including curricular change, require a school level attack. This view does not exclude the teacher but acknowledges the existence of systemic interactions within a school and their effect on developing ideas, the technical skills to implement them, and the support system necessary to sustain their introduction. The implications are clear. If the teacher wishes to effect curricular change that transcends his/her classroom, it is essential that others be enlisted as understanding allies. Since teachers tend to look to other teachers for new ideas and support, it is clearly possible for a teacher to be the initiator of curricular change.

Efforts by individual teachers to have an impact on curriculum beyond their own classroom must be mindful of two realities. Largely because of time press teachers depend very heavily on prepared curricular materials which largely determine content and strongly influence general curricular methods. The nature of public schooling tends to make the schools, and teachers, inherently conservative, best-common-practice followers. The teacher who can offer colleagues ways to chang a unit or suggest a supplemental activity that involves new content, different materials, or methodology is likely to have more success than one who proposes major revamping of the teaching experience.

There are circumstances where strident advocacy by teachers for dramatic change can be successful. Dramatic changes in education, including changes in curriculum, almost always result from external forces. Currently, the negativism toward public shcools may be so pervasive and so deep seated that those circumstances are more prevalent than usual. Teachers, individually or collectively, may be the catalyst for the design of a particular curricular change in some of those circumstances. In general, though, it is those curricular changes that can be implemented within a single classroom and that do no reduce achievement measured against locally defined performance standards over which the teacher can have the greatest influence.

An effective classroom curricular practice can occasionally be overtly spread across a department or school by a committed teacher. More likely, however, the individual teacher's impact will be confined to their work space when they modify imperfect, commonly used curricular materials and methods and make schooling come alive for pupils. The cumulative effect of individual teachers, so involved in constantly modifying the curriculum to better serve pupils, is the strength of the public school system and is the *sine quo non* of professional practice. That level of curricular change is ubiquitous and continuous, so much so that it is overlooked as the one way that the teacher can impact on the school curriculum. In an age of dramatic events in society--space shots with re-usable vehicles, assassinations, and instantaneous worldwide communication, the tendency is to consider anything less than earth shaking of no consequence. The classroom or team situation, usually removed from the front pages of the newspaper and glare of television lights, is where the individual teacher can contribute to the improvement of curriculum.

Notes:

1. Change is used in its common meaning, "to make different"--*Websters*. Curriculum may, of course, be made different in a variety of ways and degrees, e.g., from changing worksheets for pupil seatwork to change from a curriculum based totally on commercially prepared materials to one based totally on teacher made materials. It is assumed that individuals proposing change believe that the change will be an improvement. We are all aware that the "believed goal" does not always result. It is not my belief that all proposed change will lead to improvement. Nor, for that matter that a proposed change should necessarily be adopted even if it does (would) lead to a particular gain: it may lead to other negative effects that more than balance the gain.

2. Bureaucratization is used descriptively, rather than perjoratively, to mean increased formalization, complexity, and centralization of decision making.

AN INNOVATION IN ASSESSMENT: The MANS Test

Martin Herbert

An elementary school mathematics teacher is faced with a dilemma when it comes to testing. Does one use standardized tests or teacher-made tests for student assessment?

The disadvantages of using standardized tests are well known. Among those that are of concern to teachers are the following:

- The complete reliance upon multiple choice items fosters a different kind of behavior from that required in "real life" and different from most of the student's daily classroom activities. (But hand scoring of standardized tests would add exorbitantly to a school district's budget for testing.)
- Most test items aren't particularly interesting to students. Aside from the generalized achievement motivation which some students have in abundance, the motivational value seems low. Students' grades don't depend on it. They must wonder, "Why are we doing these tests anyway?"
- The economics of the testing industry require mass use of tests. They must be palatable to virtually all school districts. This tends to lead toward the lowest common denominator in the selection of content and away from lots of interesting areas that aren't universally covered. (Virtually every mathematics education group recommends increased teaching of probability and statistics, but how many standardized tests pay even lip service to them? Naturally very few, because that feature would make them unattractive to those many school districts which don't teach this topic.) There is a certain analogy here to TV programming.

- The process of reviewing available tests for both content and technical considerations is time consuming, difficult and the selection process is usually not open to most classroom teachers. (A system in which each teacher selected his or her own test is hardly feasibile either, however.)
- The usefulness of the test results is not great for most teachers. Often the results of spring testing are not available until the fall when there is a new group of students and the diagnostic value of such tests is virtually nil. Most teachers perceive, rightly, that the main purpose of the tests is accountability in successive stages: teachers to principal; principals to superintendent; superintendents to the school board; school board to parents. (But since there must be accountability of some kind built into a school district's operation, standardized tests provide one means of doing so at a very cheap cost.)

Nevertheless, there are circumstances under which such tests are useful to the classroom teacher: for instance when the instructional goals of the teacher are in accord with the back-to-basics movement. If the teacher's overriding objectives pertain to computational skills, then standardized tests provide very adequate, reliable, and well normed measures of these objectives.

For most teachers however, the acquisition of rote skills constitutes only a part of their expectations for students. Hence the need for teacher-made tests. But developing good tests is not an easy matter and most teachers don't have the technical training in test construction nor do they have available pilot groups of students with whom to try out test items.

This difficulty is not as serious in the case of testing for mastery of content; for example, the construction of a test on a unit in perimeter. This is especially true if: several teachers are involved and can pool their efforts;

there has developed over a few years a shared set of expectations of what it is students should be able to achieve; and a test has evolved over a period of time in which good items have been found and improved and bad items have been discarded.

Aside from mastering new content, there is another kind of goal, ascribed to by many teachers, which cuts across skills and content areas and which is very hard to test. It's also hard to describe since it can be described in many ways: "application", "processes", "transfer tasks", or--that most flexible of term--"problem solving".

Whatever problem solving is, there are two components that everyone would agree on. Something about the task is new to the student and the task is not an easy one that can be done blindly. The difficulty in using teacher-made tests to assess problem solving abilities is not so much in the construction of tasks (though that is not a simple matter by any means) as in the interpretation of the results. In testing for mastery, a teacher has an expectation, grounded in experience, of what students should reasonably be expected to do after instruction. In testing for problem solving, however, this expectation is affected in uncertain ways by the difficulty and newness of the problem. When a student solves 40% of one set of problems he may be doing better than when solving 60% of another set of problems. How does a teacher judge whether either performance was satisfactory?

One way to judge would be to reference the performance against other students who are similar in ability and grade level. But there are very few tests of problem solving available which have been standardized and normed in a reliable way. This is not surprising since there is no agreement on what problem solving is and it is very much outside the trend of accountability via criterion-referenced tests of mastery.

Another reason for the paucity of tests on problem solving is the lack of evidence that problem solving tasks are sensitive to instruction, i.e., it's very hard to improve kids' problem solving abilitites. The payoff is a long way off; as with goals like aesthetic appreciation, moral development, critical thinking and citizenship, one must have patience and faith.

There is no simple solution to this dilemma, but the problem has been faced by others in different circumstances, and the rest of this article will illustrate what happened in one particular case.

The Comprehensive School Mathematics Program is a K-6 mathematics curriculum development project, funded by the National Institute for Education. The program, nearing completion, is being developed by CEMREL, one of the national educational laboratories. The evaluation of the program is conducted by an independent group within CEMREL called Mathematics Research and Evaluation Studies.

The main problem in the evaluation of CSMP was how to assess the results of a program which was, to say the least, non-standard in many ways, with virtually undefined goals, many of which had to do with vague processes which perhaps tended toward problem solving.

The unavoidable question was, "How are students who are exposed to CSMP different from students who are not?" As with any program different from the norm, the burden of proof was on the "defendent" to show that CSMP students did at least as well as other students in the basic established mathematics curriculum (fairly easy to assess) and in addition, were able to do some other things better.

Now for a teacher of CSMP, observation of students over time might be sufficient evidence of the worth of the program. For a principal, the teacher's opinion plus classroom observations might be sufficient. For a mathematics supervisor, these factors plus an analysis and evaluation of the curriculum itself could be persuasive. But somewhere along the line lurks an administrator who demands, for good and sufficient reasons, "hard data" on student achievement. These data must be comparative in nature, must be fair to the groups being compared (no loaded questions), and must be credible in that what is being assessed is generally regarded as worthy.

In response to these considerations, the MANS Tests (Mathematics Applied to Novel Situations) were developed.

At each grade level, there are a series of short tests (usually about 3-10 minutes each and containing about 5-10 items) administered by a specially trained tester. For each test, the tester follows a standardized script, explaining what it is the students are to do and going through one or two sample problems.

The samples below are from the MANS tests in grade 2-6. At each grade level, about 15,000 students have been tested. The tester is a key figure in the MANS tests and the standardization of directions is important. For these samples, however, the directions are not given--just sample items--so the reader will have to assume that students will know what it is they are to do.

For convenience, tests have been grouped into categories according to the process involved. The samples on the following pages represent only about half of the various kinds of tests used. Tests involving logical thinking, pre-algebra, probability, geometry and organizing data are not mentioned at all. Often a given type of test is used at several grade levels, with increasing difficulty. For most part, only whole number problems are given, but the tests are often adapted for higher grade levels by using fractions and decimals.

Beside each sample item, the approximate percentage of students solving the item is given. Usually, an item of average or above average difficulty (i.e., 40%-60% correct) has been selected as a sample. Most of the tests begin with easy items and get progressively harder.

Estimation

In most of the MANS scales, the time allowed is of little consequence and flexible time limits are encouraged. When one estimates, however, the whole point is to do it very quickly and only to the accuracy dictated by the situation. In fact, the ideal way to test estimation skills would be to present (on a slide perhaps) computation problems, one-at-a-time, for a few seconds each and ask students to give their best answers.

One way in which estimation is tested in the MANS is similar to this technique, except that for logistic reasons, two simplifications are made. First, the student is shown a page of 6-8 problems and given a short amount of time, say 1½ minutes to complete them all. Second, the student merely has to show which of several intervals contains the answer to the problem.

One of these tests, in fourth grade, contained seven items like the following:

2 x 19	0	10	X	50	100		500	1000
40 x 10	0	10		50	100	X	500	1000
4 x 23	0	10		50	100		500	1000

Each of these items was answered correctly by about 80% of all students. Regardless of the method used to test estimation skills (and this method is no exception), one is faced with the problem of distinguishing between the student who gives a quick approximate answer after minimal calculation (i.e., an estimate) and the student who computes an almost exact answer very rapidly and chooses an approximate response.

Also in fourth grade, another estimation test consists of a series of 12 items, like the one below, which are to be done in about 3 minutes. The student must put either 2, 5, or 10 in the blank, whichever is the best estimate.

100 is about ____ times as large as 19. (50%)

Producing Multiple Answers

Most problems that students work on in mathematics have one correct answer. One kind of MANS test presents students with problems for which they can generate many correct answers. Two kinds of scales are used.

The first is what might be called a "fluency" scale, requiring students to produce as many of the limitless number of answers as they can in a given amount of time. For example, in second grade, after seeing and discussing a few ways of writing number sentences about 9 (9 = 10-1, 9 = 3x3, 9 = 1+5+3, 9 = 18÷2) students are asked to show as many number sentences as they can about 8.

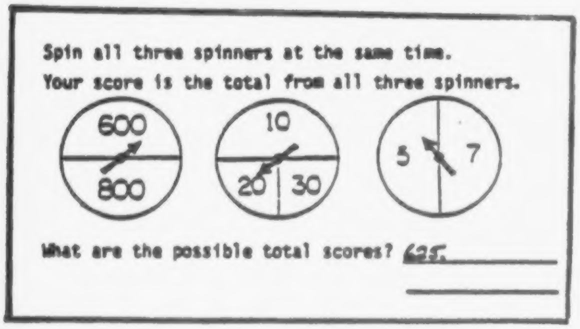
8 = __________

8 = __________

etc.

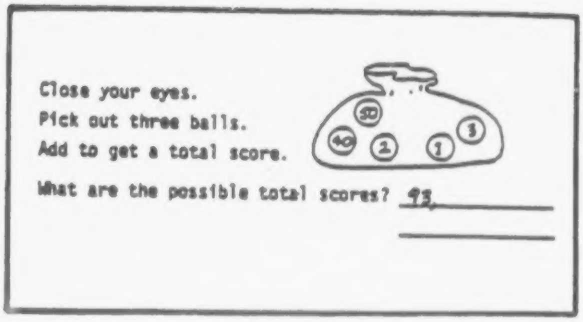
The average "score" (number of correct answers minus number of incorrect answers) for second graders was about 6 (with a limit of 4 minutes).

The second method of testing the production of multiple answers is to pose a series of 3 to 5 situations, each with a question to which there are several correct answers. The type of situation posed is illustrated by the two problems taken from sixth grade:



Spin all three spinners at the same time.
Your score is the total from all three spinners.

What are the possible total scores? 625.



Close your eyes.
Pick out three balls.
Add to get a total score.

What are the possible total scores? 93.

The average score (number right minus number wrong) was about 6½ out of 11 on the first problem and about 5 out of 9 on the second problem.

Mental Arithmetic

The directions for this type of test are in essence quite simple, in effect: "Do the problem but don't do any scratch work."

The problems are not meant to test a student's ability to do complicated calculations in one's head. Rather they are problems for which the easiest and most natural way to get the answer *is* purely mental. i.e., without scratch paper.

For example:

1. ☐ - 200 = 100 2. 3 x ☐ = 300	second grade
3. 375 + ☐ = 600 4. ☐ x 250 = 500	third grade
5. 700 - 401 = ☐ 6. 150 divided by 25 = ☐	fourth grade
7. 4,999 + ☐ = 10,000 8. 25 x 32 = 800 26 x 32 = ☐	fifth grade
9. (8 x 29) + (2 x 29) = ☐ 10. 1200 ÷ 30 = 40 1200 ÷ 15 = ☐	sixth grade

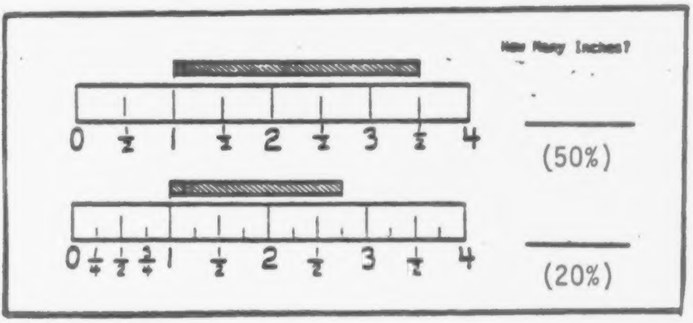
For each of the above items, between 25% and 50% of the students did the item successfully. The items are indeed computationally oriented, some more than others, but it is clearly a different kind of computational alertness that is required.

Representation of Numbers

Sample 1, second grade:

What number is 10 more than 95?_____ (32%)

Sample 2, fourth grade:

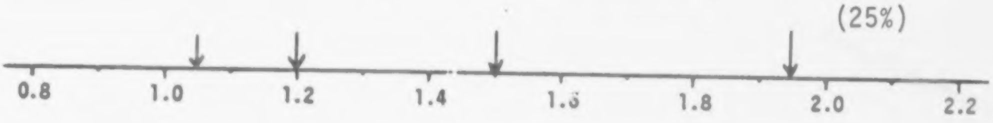

How Many Inches?
0 ½ 1 ½ 2 ½ 3 ½ 4
0 ¼ ½ ¾ 1 ½ 2 ½ 3 ½ 4

______ (50%)

______ (20%)

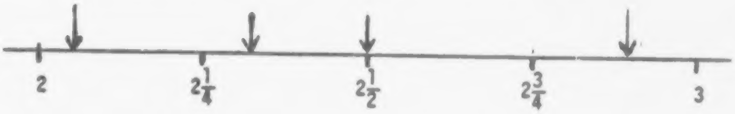
Sample 3, sixth grade:

Circle the arrow that points to $1\frac{1}{2}$ on the number line below.


0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2

(25%)

Circle the arrow that points to 2.05 on the number line below.

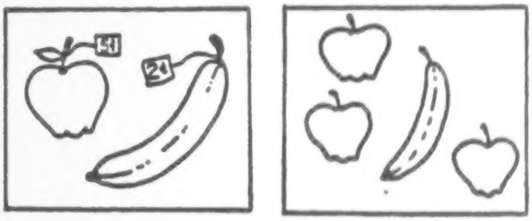


(65%)

Specialized Word Problems

There is a deliberate attempt, when assessing students' abilities to solve various types of word problems, to reduce drastically the reading load and the computational level of the problems. For instance, the problems are read, one at a time, to the students, in second and third grades.

Second grade:



Apples cost 5¢ each and bananas cost 2¢ each.

Sally buys 3 apples and 1 banana.

How much does it cost altogether? ________

(50% correct)

Two Stage Word Problems (third grade):

Jim has $10 in his bank now.
Each week he will save $5.
In how many weeks will he be able to buy a radio that costs $30. ________

(35% correct)

Extraneous Data (fourth grade):

Sue has 12 bottles.
It takes 36 bottles to fill a case.
It takes 6 bottles to fill a carton.
How many cartons can Sue fill? ________

(60% correct)

Three Stage Word Problems (fifth grade):

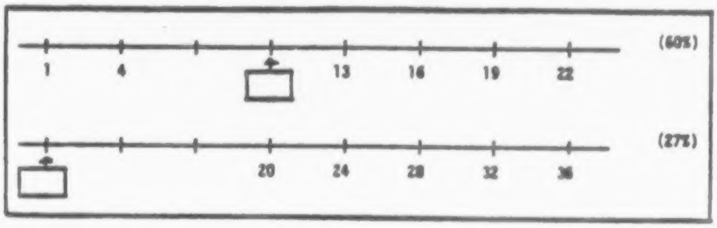
Joe puts boxes into piles.
Each box is $\frac{1}{2}$ foot high.
Each pile is 5 feet high.
How many boxes does he need to make 3 piles? ________

(35% correct)

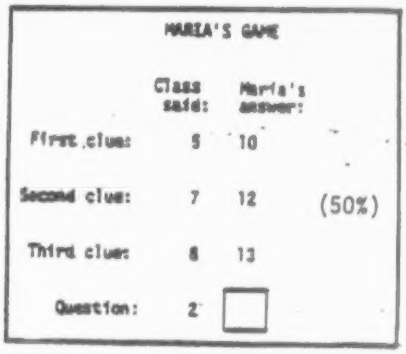
Number Patterns and Relationships

Each of the samples below requires rather extensive directions and sample items for the students to understand what the "game" is.

Sample 1, from second grade:



Sample 2, from third grade:

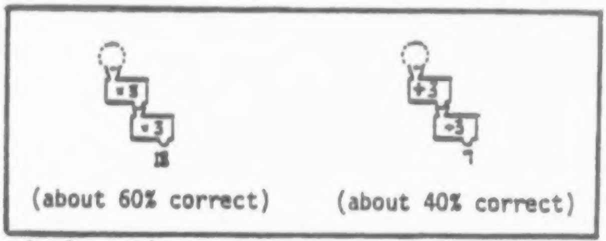


MARIA'S GAME

	Class said:	Maria's answer:
First clue:	5	10
Second clue:	7	12
Third clue:	8	13
Question:	2	☐

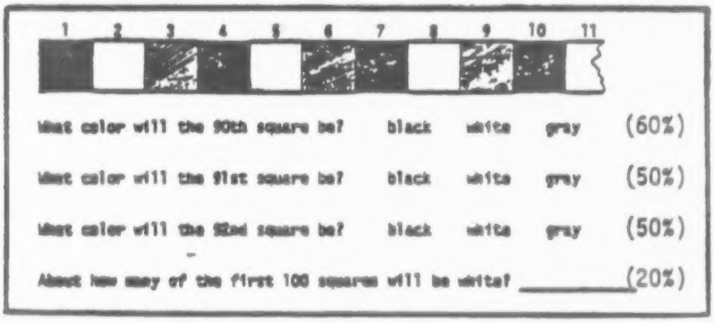
(50%)

Sample 3, from fourth grade (complete the dotted circle, i.e., the "input"):



(about 60% correct) (about 40% correct)

Sample 4, from sixth grade:



What color will the 90th square be? black white gray (60%)

What color will the 91st square be? black white gray (50%)

What color will the 92nd square be? black white gray (50%)

About how many of the first 100 squares will be white? ________ (20%)

An information pamphlet about the MANS Tests can be obtained from the author by writing to:

CEMREL, Inc.
3120 59th Street
St. Louis, MO 63139

Also available are evaluation reports describing the strong performance of CSMP students (versus Non-CSMP students) on these tests.

ON ASSESSMENT

James R. MacLean

Let me start with some background. The Province of Ontario has approximately two million students and 100,000 teachers in its educational system. Geographically, it is bigger than Texas. From Toronto it is 1400 miles to the furthermost western school, 900 miles north and 300 miles south. But there is another important element. Ontario has the longest continuing government in the western world. We've had the same political party in power for 38 years. This has provided a continuity that has influenced the goals we have set for ourselves. Stability in government does not mean that policies do not change. My first five years, beginning in 1966, were tremendously challenging and exciting. This was when Edith Biggs and I worked together. We were able to bring in people who made a very valuable contribution to our system. They were the golden years. Since then, we've become a little more parochial, additional rules have come to apply, and the same resources aren't available.

The Ministry was reorganized in 1972 and, from my perspective, the changes were not for the better. Then it was reorganized again in 1975 and repeated in 1977. Things went from bad to worse. I was assigned to the Research and Evaluation Branch. I thought, "This is the end. I have to find another job." Now I say, "Folks, you're looking at living proof that even adults don't always know what's best for them." In the five years since I've been in evaluation, I've been able to do more to protect curriculum than I ever could have done if I'd stayed in the Curriculum Division. Sometimes it is worthwhile enduring the agonies of working in a huge bureaucracy.

When living in such close proximity to a very powerful neighbor, our leaders tend to always cast their eyes southward when they are looking for innovations in education. Our Minister was no exception to this habit. Professor Varga commented on the good things in your system. I must say we owe you an enormous debt as well. In 1976 our Minister of Education, hearing of the developments there, decided he was going to come down and talk to the politicians in some of the places where they had very elaborate evaluation schemes. He took along my boss to talk to the educators. At that time we were planning to test every child in the province in grades 4, 8 and 10 in mathematics, English (or French) and physical fitness. We even had drafts of report cards that the Minister was going to use to report directly to parents. This was what politicians felt would satisfy the critics of the Ministry of Education which was spending a great percentage of the provincial budget.

Well, he went to California, Florida, Michigan and Washington. And, almost without exception, the politicians told him, "Look. If you're not into statewide assessment programs, don't. If you think they will satisfy your critics, forget it." He came back to Toronto and scrapped the plans to do province-wide testing. For that, America, I thank you.

We then undertook some extensive research which included studies that looked at how well the grades assigned to students leaving the school system predicted success at university. (Incidentally, in the Province of Ontario we have had no external examinations at any level for something like 15 years. The only credentials that the students are able to offer to the colleges and universities are the grades assigned by the school when they leave.) Our researchers compared these grades with those of the College Entrance Board, the Oxford-Cambridge "A" Levels in Great Britain, and our former Honour Graduation examinations and found that they were better, more accurate predictors than formal examinations. The policy on assessment in the Province of Ontario at the moment reflects this outcome. Because the teachers are doing a very good job, the government will provide the resources that will help them do a better job. The project is called the Ontario Assessment Instrument Pool. It is a pool of assessment instruments directly related to the curriculum which will be accessible to students, to teachers, to principals, to local board officials and to Ministry personnel. We can all dip into the same pot. There are no hidden questions or secret items. There are also no tests. Each item is developed by teams of teachers and measurement experts, field tested and approved by a Subject Advisory Committee before it is published. It is my responsibility as the supervisory official.

I'll come back to assessment in a moment. -Let me digress - Professor Varga talked about the fragmentation of the curriculum during our seminar. I recalled an experience when I was Assistant Superintendant of Curriculum for the Ministry. As chairman of the largest curriculum revision operation ever mounted in the province, I tried to eliminate the traditional subject discipline headings. For years it was the goal of the school system to have students ready for the university. The curriculum was built from the top down. It was always a getting-ready-for process. Secondary schools were getting students ready for the university, elementary schools were getting students ready for secondary school, and the parents' job was to get their kids ready for school. With fewer than 20% of high school graduates going on to university it seemed ridiculous to have the entire program dominated by the entrance exams. Instead of getting kids ready for school, we should get the schools ready for kids. Now that sounds great, but how do you turn things upside down without chaos?

Something's wrong when you see parents striving to teach their kids the alphabet and to count to 100 so they will be all ready for school. As a result of our Ministry curriculum commission, we eliminated mathematics. Actually, at one stage we were on the verge of having no subject disciplines in the elementary school, which is junior kindergarten (4 years) to grade 8 (14 years). We thought that was going a little too far and so we ended up with three subject areas. One, called the Arts, included music, drama, physical education, etc; another called Communication included mathematics, language (English or French), etc.; and the third, Environmental Studies, included all the rest. Actually, Environmental Studies was the foundation upon which the other two rested because it was based on experiences which came from the environment. Given the experience, children expressed their feelings and understandings through the arts and through communications. Well, the mathematicians in the community were ready to have me scalped and my head hanging outside the Legislature because math was going to hell in a handcart--MacLean had wiped it out of the curriculum--little realizing that instead of having that 40 minute slot per day for mathematics, they could have five hours if they wanted. Balance in the curriculum was not achieved by time schedules for each day: it was achieved over a longer period of time, the rationale being that kids don't fragment their experiences into "Now I'm doing math, Now I'm doing language, Now I'm doing music, Now I'm doing this, Now I'm doing that."

Back to evaluation, now. There is a second project. The other area of evaluation that I'm concerned with concerns children coming to school. I'm working to develop instrumentation in this field. Traditionally in my province, early identification of learning disabilities and the prescription of the kind treatment these kids needed and the placement where they would get it. My first action was to supervise a research project designed to survey the province on what practices were in effect, what school authorities were contemplating, and why they were doing so. Disabilities was changed to abilities and the second thing was to get early abilities assessment taken from out of the Special Education Branch because it has nothing to do with special education. If schools are ready for kids, then we need to know something about the nature of kids when they come to school. The Ministry contracted a research study to learn about children's characterisitics upon school entry. We did another study on parents' expectations of school and a third on what does school expect of kids. Now we're in the process of trying to fit these things together and perhaps evolve programs that will reflect what students really need.

Now I know all this sounds very grandiose and straightforward and it sounds like Ontario is the distant land where everything is perfect. I hasten to assure you that it is not. It's only when I come to St. Louis that I look back and say, "Gee, it looks good," but here I don't have all the daily problems to face.

Back to the Ontario Assessment Instrument Pool. For the purposes of this pool, we're saying that assessment is the process by which you gather information. Assessment can include a great variety of tests and, as we do, it can place a heavy emphasis on observation. Another research study found that standardized tests were used rather extensively but that the teachers weren't very satisfied with them because they didn't fit the curriculum. They also thought that there were a lot of tests that weren't very useful. We already knew that teachers didn't have very much confidence in the tests they designed themselves. They relied largely on observations but they had little or no confidence in the judgments they made based on what they saw. Now, the Ministry is spending a lot of money developing instruments and strategies to help teachers improve their observation skills, to know what to look for, to know how to sort out that information and how to interpret it, and then to decide what to do about the things that they see.

Observation is one of the very major features of the pool.

In this connection, we are developing implementation strategies for principals. Should principals decide that improving the observational skills of teachers is a priority, we'll provide them with three or four different techniques. If they budget five hours of in-service they can do one kind of program. If they have fifteen hours, a different program can be employed using other strategies. We also have instruments that distric school administrators can use if they want to develop a program for improving the observational skills of teachers.

Assessment is the collection of information. Evaluation is the judgment based on that information. Evaluation is the decision that is made by the teacher, by the principal, by the administrator. It cannot be made mechanically. It cannot be done by a computer. The evaluation is the judgment by the human in the situation. Such a view is contrary to much of the literature and there's a lot of misunderstanding if we talk about evaluation as being some kind of mechnaical process. It isn't mechanical. You must distinguish between assessment (information gathering) and evaluation (judgment).

One of the problems in any province or state system is the difficulty of trying to convince the teachers' organizations that there isn't someone somewhere in an ivory tower that dreams up these great schemes and ships them out. I've never believed that you can change people with print but I have been unable to convice the government of that. The commitment to paper (bulletins, memos, etc.) is about the greatest educational weakness in North America generally and, in particular, in Ontario. On previous visits here I described the golden years when I worked face-to-face with groups of a hundred teachers at a time for 5-6-7 days to convince them of the practicality and reasonableness of our curriculum approach. Now I'm restricted. I sit at Queen's Park (Ministry offices) and write all these missives. We throw them up like seeds and hope that they land on fertile ground. (If you know our province, about 40% is water, another 40% is rock. So we have 20% fertile ground--and not all of it is cultivated by any means.) The biggest weakness of this process is that we produce this material, send it out and wait for something to happen. This situation is in contrast with Great Britain, where at least 50% of the budget for most educational projects is for teacher in-service. They don't believe you're going to change people just because they read something in a book.

To overcome the notion of the ivory tower, and to convince teachers that this is worthwhile, we made sure that every instrument pool has many authors. Rathern than reinventing all the materials, we gathered instruments from school authorities and from every source we could find. If we could bring an instrument up to the standard that was required for the pool, we would use it. I mentioned that it is an open pool. Every instrument that is included and published has been approved by subject advisory groups made up of representatives of teachers and school authorities. (I didn't mention one of the characteristics of Ontario schools is that we have had two school systems ever since our early days--a public system from junior kindergarten to grade 13 and a Separate system, the Roman Catholic Separate Schools, which at the moment extends to grade 10. We hope it will extend to grade 13 as well.)

Compulsory subjects were reintroduced to our secondary schools in 1977. These subjects provide a focus for the early development of materials for the instrument pool. At the secondary level, the compulsory subjects are mathematics (2 credits), English or French (2 credits), Canadian history and geography. My responsibility at the moment is mathematics--grades 4-10--and early identification.

There are two basic purposes for the Ontario Assessment Instrument Pool. One thing we have to emphasize very frequently: it is not an assessment of teaching performance. The Ministry provides curriculum guidelines in various subjects (elementary doesn't have individual subjects--we have guidelines for the total program), and so we're interested in knowing how our guidelines are working. This we call "program evaluation." (In order to accommodate regional differences, our curriculum guidelines are just that--guidelines. It is required that local authorities develop their courses and programs using principles and objectives expressed in the Ministry documents. They have no choice in this matter.)

(I'm not going to go into the whole history but at one point the minister had at his disposal something like 500 inspectors. Each of them had a certain section of the province and they inspected them regularly, rated the teachers and reported to the minister. The year I joined [1966] the Ministry called them in and told them that the inspectorate had been abolished and that they would become program consultants. They were now appointed by the minister to help boards develop curriculum at the local level.)

We are concerned about how well the programs are working. And school authorities are concerned about school achievement. These are the two components of the assessment operation. The instruments can be used by teachers for identifying the needs of students, evaluating their achievement, reporting, reviewing standards, etc.

There's a great deal of anxiety on the part of teachers about evaluation. Perhaps the most notable success lately came from the first of the instrument pools, mathematics for the intermediate division. When teachers got a look at that and saw there were no surprises, no esoteric, difficult questions, etc., they heaved a sigh of relief and said, "We don't have anything to worry about." And they don't. What they do have to worry about is if, as is traditionally the case, they are not teaching the geometry component of the program they'd better worry, because 40% of the instruments in that pool are related to geometry. Our view is that mathematics is not only concerned with numbers.

Since this publication will be read by teachers and school administrators, let me ask you a question. How do you assess school achievement? Do you use, for example, scores on tests whether they purport to be diagnostic or achievement or whatever? I have mentioned the difference between assessment and evaluation. The literature talks about diagnostic tests. I say there's no such thing as a diagnostic test because a diagnosis is a judgment and a judgment can only be made by an individual. How can you have a test diagnose anything? Think about that!

Permit me to digress again for something important. I think the role of the school principal is utterly critical. As I should have mentioned before, the qualification for principal at the school where I was principal before going to the Ministry (the King Edward School) was to be physically stronger than the kids. I didn't want the school. I had a much newer building in a wealthier section of the city and to be arbitrarily assigned to the other side of the tracks was a blow to me. The school had been organized by my predecessor. We had 700 kids from kindergarten to grade 8 in an old building, built in 1900, three floors, where the kids were accustomed to being lined up in rows by the teachers. If your class was quietest you got to march in first while they played martial tunes on the P.A. system. Teachers were stationed in the corridors--no running, no talking--all that. Kids entered the school through one door, the back one. Well, you can imagine how long it took to get 700 kids in--morning, morning recess, noon, afternoon recess. They were wasting most of the day lining kids up and marching them in and out. So the first thing I did (a new principal with lots of teachers older than I who knew the community and were pretty resentful of this new hotshot) was to say at a staff meeting, "We use the front door for the teachers and the back door for the kids. Is there any reason why we can't let some of these kids come in the front door?" The only reason was, "We've never done it that way." So I said, "Starting Monday, we're going to do it and we're not playing marches on the P.A. system either. When the bell rings, we'll experiment with letting the kids come in of their own free will." Of course we cut the time by about 80% and the teachers were astonished that the kids could take some responsibility for entering the school in good order. That was just the first step.

There was a class--there always is a class--that didn't fit anywhere. In this case we had a class made up of very young bright girls and much older slow boys. (Do you have classes like that?) This was about grade 3-4 and believe it or not I had boys of 15 years of age who had never passed. They spent two years in kindergarten, two years in grade 1, two years in grade 2--social promotions--and then there were bright girls who maybe didn't even bother with kindergarten but came straight to grade 1. They ranged in age from 15 down to about 8. Talk about the one-room rural school! None of the staff would accept that class, it was such a challenge. The boys had terrorized that school. I had a new vice principal at the time and I said, "Norm, we will share that class." We moved it next to the office and I said, "You take the morning and I'll take the afternoon." I ended up teaching mathematics, science, music and art.

You know that when there is trouble in the school, it doesn't matter who is in the office, they always come looking for the principal. They kept hammering on my door even though the vice principal was in the office. One day somebody had stolen a bicycle at noon. It had disappeared and we had a crying little 8 year old. About 3 in the afternoon, someone found the bike. It had been damaged. Knock, knock, knock on the door. This was very irritating. At the time, I was a pretty run of the mill traditional teacher. Without thinking, I said, "Bring that bike in here." I didn't want to trot outside to look at it. They wheeled the bicycle in. All of a sudden for the first time in their school careers, those 15 year old boys knew more about the focus of attention than anyone in the room because they'd put bikes together and torn them apart a dozen times. Well! It clicked up here and I said, "Hmmmm." To make a long story short, I'll tell you that we used that bicycle as the focus for math, music, art and for science for aobut three months. You wouldn't believe it! I almost didn't believe it myself but gradually I thought, "If they're interested in the bike, maybe they're interested in other things." The next thing that attracted them was a stop watch. We had some planks and bricks and toy trucks around and when the kids finished their regular work, they went to explore this material. I'll shorten the story a lot. It started with two boys--Peter and his friend. (Peter was the biggest hellion in the school, IQ less than 80 according to all the tests: his father drove a bus in the city.) So Peter and his friend got a plank and put it down and put a brick under one end of the plank. They started rolling the toy cars down the plank and they timed them. All I insisted on was that they record in some way what they did and what they found out. So they thought, "His truck went down faster than mine." They developed all kids of strategies. They had blocks at the top that pulled out and another row of blocks at the bottom to make sure that they could time it as accurately as possible. They did a number of trials with lots of addition, averaging, arithmetic, etc. They didn't care about that. They wanted to know about the performance of the vehicles. One day Peter said, "Sir, if I put two bricks under the plank, will it come down in half the time?" What would you say? I said, "Try it, Peter." Well, he did and he ended up with the plank nearly vertical. Finally Peter said, "Sir, I don't want to do this anymore. I've got a problem of my own I want to investigate. (80 IQ--remember?) In their recording they had a whole lot of big one inch squared graph paper. They had made a nice little graph. With the big squares underneath Peter said, "As the slope (believe it or not, the word slope) of the graph is changing, the area under that slope is also changing. I want to find the pattern." With that, of course, he left his teacher far behind. I said, "How do you know there's a pattern?" He said, "Sir, in mathematics there's always a pattern. You only gotta look for it." Well, Peter looked for it--took him six months--but step by step, he retraced the path of Archimedes 2000 years before and worked through to the integral and differential calculus. 80 IQ!

Now, I don't know how many Peters there are in our school systems, but I know this one Peter was enough to change my whole philosophy about education. It wasn't just Peter, but the whole class actually. I'd suggest that there is a great deal of infantilization of the school program. We spend so much time on trivial things requiring them to be mastered to such an idiotic degree that kids get turned off. They get absolutely destroyed in terms of their mathematics education.

Back to assessment. Here are some of the assessment instruments. [See Appendix.] Some are pretty traditional. A lot of them are not pencil and paper. They often require an interview where you sit down and discuss the issue. We have a number of these folders about basic assessment--what to do, what may happen and, on the back, we have suggestions for children who need help. In other words, you make the diagnosis and you prescribe the remedy using the best judgment that you can make for what will work.

Now, people may say, "What does that have to do with the basics--being able to do multiplication, division, addition, etc.?" I suggest it has a lot more to do with the basics than what we usually test. So often we dismiss mistakes. We cannot, in my opinion, use the traditional test materials to diagnose, to evaluate, to make a judgment because we usually don't have enough information. I observed a girl, age 11, who on a test had written:

$$1/2 \times 1/3 = 1/6$$

What would you do if you were marking this? Make a check mark? I said, "Which one of those is the largest?" Without hesitation she said, "That one," and circled the 1/6. "Why?" I asked. "Well, when you multiply, you always make things bigger." If she maintains that fundamental error in her thinking, she's going to encounter all kinds of difficulties. But on the test of basic skills, she had scored 98%. The same thing happens with older students. This man was 35. He wrote:

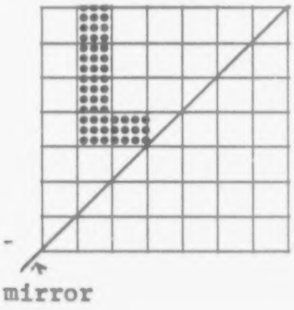
$$.1 \times .1 = .1$$

Now he knew that this mean $1/10 \times 1/10 = 1/100$. But he was troubled with the idea that multiplication always made things larger so the only solution he could come to was to compromise and write $.1 \times .1 = .1$. It was the only solution that satisfied both of those conditions.

I suggest that if we're going to go about this business of assessing and evaluating, we should get to the basics. We should get to the fundamental ideas about mathematics or whatever issue it is. For a long time I've been trying to convince our teachers that we should never start mathematics learning with arithmetic. (That's going to be too big a change to make without chaos. Parents' expectations are that children starting school should know how to count and if they can count, they're well on their way to success. The problem is that when kids come and they can recite the number names in order--even to 100--we assume that they can count. Recitation of number names has very little to do with counting and understanding number. The real difficulty occurs probably about grade 5 or 6 when the absolute misunderstanding about the fundamental nature of number is revealed. But as a teacher you couldn't believe it. You couldn't believe that at this age they didn't know what number was. And you don't remediate or assist with the critical point in the difficulty. You keep on patching things up. If you can't divide fractions, what's the solution? Divide more fractions. You keep going, hammering away until they devise a certain facility through rote memorization that doesn't endure. Any teacher can tell you. She gets kids in grade 7 who at the end of grade 6 could do a particular skill. But now she says, "What did they do in grade 6 last year. The can't do the fundamentals.") I think it is important to deal with basics, identify the basics. The real basics. For me, geometry is the best starting point for young children.

Here's one item that I found very interesting. Along with each instrument, incidentally, we're publishing what we call facility levels--not difficulty levels but facility levels. Of every 100 students that did this question, a certain percentage got it right. We're also doing a lot of other analyses of the instruments, relating them to specific objectives in the guidelines. This problem had a facility level of something like 18%. That is, 18% of the students got it correct. But when we took the mirror line and made it vertical, we had 83% facility.

Draw the reflection.



Following is the history of the OAIP, then some sample items.

One final word. When the opposition critics say that the quality of the students in the schools is declining, when our university professors stand up and say, "The students today don't know half as much as they did 20 years ago," we'll be able to answer the critics with data. When we are interested in how well our program in calculus is meeting the purposes for which it was designed, we then sample the province--sample in such a way that individual students are not identified, classes are not identified, schools are not identified, districts are not identified. We need a small sample to give us a reliable indication of how our grade 13 calculus course is going so we can guarantee the anonymity of every one of those stakeholders in this project. The same is true if we want to find out how our students at the end of the elementary school are doing in language or history or whatever. As far as the government is concerned, it is a sampling process and we will exert all the pressure we can to not allow school authorities to administer tests to every student in every grade with the sensational publicity that surrounds the ranking of schools, etc.

At the moment, that's the way it is. Thank you.

of
Education
Ontario

THE ONTARIO ASSESSMENT INSTRUMENT POOL
A CURRICULUM-BASED AID TO EVALUATION

For evaluation to play a positive and constructive role in education, it must serve the needs of many people within the education community: students, teachers, school administrators, board officials, trustees and officials of the Ministry of Education. Because all these people have a stake in the evaluation practices being conducted throughout the province, they are all entitled to participate in and benefit from a consistent, just and responsible evaluation policy.

The stakeholders, however, have a variety of evaluation needs. The Ministry, for example, needs the capacity to evaluate provincial programs, as the basis for improving its policies and responding to its obligation for public accountability. School board officials, administrators and trustees have a similar need with respect to local policies and programs, supplemented by the need to ensure the effective evaluation of individual student achievement. Students and teachers, at the very heart of the educational process, need to know on a more personal and individual basis how successfully they are accomplishing their goals. All groups have a need for a clear, forthright and convincing method of reporting to others the results of their evaluation procedures.

As a result of considerable investigation, study and research, both inside and outside the Ministry of Education, several convictions have emerged. One of these is that there is a distinction between evaluation and assessment. The primary purpose of evaluation is to make judgments that will help to improve the quality of education; the primary purpose of assessment is to provide data on which the evaluative judgments can be based. Unfortunately many evaluative practices serve neither to improve educational policy nor to stimulate and extend student learning. There is little evidence, for example, that the mere introduction of large scale assessment programs to allow simple annual comparisons of student or school performance can assist students, teachers or administrators. At best the effects seem neutral; at worst, harmful.

Another conviction, growing out of the first, is that any assessment program must include not only the methods of gathering data but also the techniques for analyzing and communicating the results. Techniques must be developed so that the information that has been collected can be used wisely and efficiently to improve educational processes, whether these be individual student programs or the broad provincial policies that support and regulate such programs.

A further conviction, based both on widely held opinion and on the results of formal research, is that the assessment instruments currently available are not good enough. Even the best standardized tests relate well only to a limited number of the objectives being pursued in Ontario classrooms today. There are few reliable methods for assessing such matters as problem-solving skills, creativity, moral development, self-concept, attitudes to learning, concern for others, and a host of other objectives that Ontario schools have long been striving to achieve.

Given the framework of provincial needs and the foregoing convictions, it is clear that one of the first steps that must be taken is the provision of a wide variety of assessment strategies that have been carefully prepared, tested and refined to correspond to the goals and objectives of Ontario education. This project has now begun. A major program has been initiated by the Ministry of Education to obtain the tools necessary for first-rate evaluation.

The program is called the "Ontario Assessment Instrument Pool", not a particularly neat or glamorous title, but one in which every word conveys a precise intention: "Ontario" indicates that the pool will contain only instruments that are relevant to Ontario schools; "Assessment" emphasizes that the methods and instruments collected will be appropriate for gathering data for assessment and measurement rather than for evaluation, which is the result of human judgment and not the mechanical outcome of the technical machinery; "Instrument" carries a broad connotation to indicate the wide range of tools that will ultimately be available in the pool; "Pool" itself implies a sharing, something to which many can contribute and from which many can draw.

The Ontario Assessment Instrument Pool is being developed collaboratively. Although the Ministry of Education provides the leadership and most of the funding, school boards, teachers' and officials' organizations, subject groups, faculties of education, and the Ontario Institute for Studies in Education are cooperating to collect existing materials and to help develop new ones.

It is expected that the wide variety of assessment methods and instruments contained in the Pool will serve two basic functions: to assist program evaluation at the provincial and local levels and to assist the evaluation of student achievement at the classroom level for both diagnostic and summative purposes.

Eventually, the Pool, along with the methods and specifications for further instrument development, will be placed at the disposal of the educational community. All of the instruments will be made available to anyone wishing to make use of them.

Research contracts to develop these instuments were let in two stages. Those in mathematics/mathématique (Grades 7-10), English (Grades 7-10), French as a second language (Grades 6-10), history (Grades 7-10), and geography (Grades 7-10) terminate at the end of April 1980. Instruments resulting from these contracts will be available to the schools in the Fall of 1980.

The second group of contracts for mathematics/mathématique (Grades 4-6), anglais (Grades 4-10), français (Grades 4-10), chemistry/chimie (Grades 11-13), physics/physique (Grades 11-13), histoire (Grades 7-10), and géographie (Grades 7-10) terminate a few months later. These instruments will be ready for use in the Spring of 1981.

Each contract team includes groups of teachers and other board personnel, many of whom have already been engaged in program development and evaluation activities related to various Ministry guidelines. As the resulting assessment instruments are examined and refined, they will be made available for field trials. At the same time they are being reviewed by a Subject Advisory Group, one for each contract, whose responsibility is to ensure relevance to curriculum objectives as defined in Ministry guidelines and elaborated by local school boards. Each Subject Advisory Group is chaired by a curriculum specialist of the Ministry, with membership drawn from Ontario Teachers' Federation representatives and other subject experts across the province. The Principal Investigator for each contract and the Ministry education officer supervising each contract are ex-officio members of these groups.

The foundations for the Ontario Assessment Instrument Pool are being laid with great care and deliberation. It is only after ensuring technical quality in the instruments, indexing and coding them in relation to guideline objectives, describing their measurement characteristics such as level of difficulty, reliability, etc., and determining the best methods of accessing and disseminating them to users that the developers of the Pool can have confidence in the quality and usefulness of their products.

How will the Pool be used? Several patterns of implementation are possible. Teachers can draw instruments to assess the achievement of their own students in relation to classroom objectives. Data from this assessment can be used by teachers to help form a basis for informing students and their parents of individual student progress or for making judgments about the suitability of classroom materials and activities. On occasion teachers may also have a need for information to match internal standards with widely accepted external standards as reflected in data already gathered from a provincial or regional sample survey. In fact, teachers are constantly engaged in one way or another with such evaluative processes. The Ontario Assessment Instrument Pool will provide a source of appropriate and reliable instruments to assist these processes.

Another pattern of implementation might be adopted by principals interested in looking at the degrees to which program objectives are being met or how student achievement is being assessed and communicated. With the concurrence of the staff about the nature of various program objectives, it would be possible to draw from the Pool a number of different instruments to be administered either to whole classes, to whole grades or to a sample of students selected randomly from across a grade. Information derived in this way could help the principal judge the appropriateness of the programs and levels of achievement within the school and provide an informed basis for improving instruction.

Implementation of the Pool can also be undertaken on a larger scale by school boards and the Ministry of Education. To help boards discharge the responsibilities delegated to them, evaluation at the local level must assist the rational development and improvement of local policies and the allocation of resources. Program evaluation can be assisted by administering instruments from the Pool to randomly selected students. Data from such a sample survey could provide precise estimates of the performance of a total group of students in relation to the particular objectives being assessed at any one time. This data could then help those making decisions about the particular needs of the system with regard to program development and resources.

It is important that as provincial policies and programs are introduced, the Ministry of Education have methods of monitoring their effectiveness and making judgments about what further developments or refinements are necessary. These judgments must be based on information that is accurate, relevant and timely. To be accurate, the information does not have to come from all schools in the province or from all students in any one school; it can be derived from a representative sample whose performance will provide an authentic picture of general achievement. To be relevant and timely, the information must be gathered by methods that truly reflect current educational policies and programs in Ontario. These methods will be available in the Ontario Assessment Instrument Pool.

It should be understood that the main aim of the Ministry is to develop and make available a collection of good assessment instruments that may be used to help evaluate some aspects of student performance or program effectiveness. These instruments will not supplant most of the assessment techniques that are already being used in classrooms across the province. They will supplement them by providing sound, carefully constructed, curriculum based instruments.

It is not the intention of the Ministry to use the Pool to compare the performance of individual students, schools or boards. Such use of the Pool would be contrary to the intent of its sponsors and developers and prejudicial to its further development and use.

It is, of course, the intention of the Ministry to evaluate thoroughly its own provincial programs and policies. Any such evaluation would inevitably include some assessment of student achievement, probably through a representative sample of students from across the province. In whatever sampling procedure is used, the Ministry is committed to preserving the anonymity of students, teachers, schools and school boards.

Finally, the provincial evaluation policy and the assessment program available through the Instrument Pool will themselves become part of the self-evaluation in which the Ministry is engaged and will in their turn be scrutinized and reviewed for their effectiveness and validity. In this way the Ministry will continue to discharge its responsibility to provide sound policies and programs to meet educational evaluation needs in Ontario.

For further information on the Ontario Assessment Instrument Pool and related research projects see:

Review and Evaluation Bulletin, Volume 1, No. 1, 1979: "The Ontario Assessment Instrument Pool: A Curriculum-Based Aid to Evaluation".

Copies may be obtained at $1.50 from:

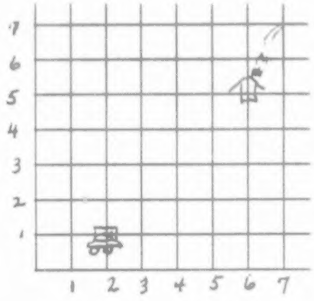
The Ontario Government Bookstore
880 Bay Street
Toronto, Ontario
M7A 1N8

(Cheques should be made payable to
Treasurer of Ontario)

or from:

The Ontario Institute for Studies in Education
Publications Sales
252 Bloor Street West
Toronto, Ontario
M5S 1V6

A fire truck is located at point (2,1). It must get to the burning house at (6,5). Use points to describe 5 routes it might take.



Use graph paper. Outline 5 squares to form a figure. Follow these rules:

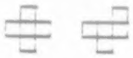
1. all squares must form 1 unit
2. each square must touch along the exact full edge of at least one other square.

Make as many different figures as you can.

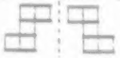
Related activities

Have students:

1. use 3 squares and find all solutions (repeat using 4 squares)
2. find all 5 square solutions which result in a map which can be folded to form a box (two such solutions are as follows)



3. find all possible solutions to this challenge
4. find the perimeter of 10 solutions (discuss the idea that area does not conserve perimeter)
5. draw pairs of solutions which are reflections of one another



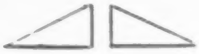
6. find solutions which have an internal line of symmetry

Multiply 5 pairs of 2 digit numbers. What rules can you make about the products you obtain?

Related activities:

Have students:
1. change the identified set (odd numbers, even numbers, 2 digit multiples of 3...)
2. limit the set (2 digit numbers beginning with 5)
3. investigate series within the set to see what happens to answer size (i.e. 10X99=990, 11X98=1078...)
4. repeat challenge using one 2 digit and one 3 digit numeral.
5. attempt to find pair which will provide a particular given product (write two 2 digit numbers that provide a product of 359)
6. repeat challenge using 3 digit numerals

Match equal edges of two 30°, 60°, 90° triangles. Trace the different arrangements you can make. How many different figures can you discover?



Related activities:

Have students:
1. identify figures produced
2. develop logical account to justify number of figures found
3. find and sort symmetrical and non-symmetrical digures found
4. find perimeters and areas of each figure formed. (explores possibility of conservation of perimeter by area)
5. explore slides, flips and turns as they relate to this task

Which polyhedron illustrated below could be constructed from the net in figure 1?

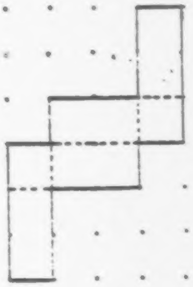
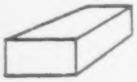
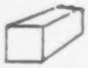


Fig. 1

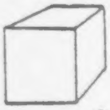
(A)



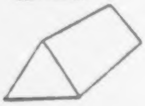
(B)



(C)



(D)



Which of these nets can be used to construct a cube?

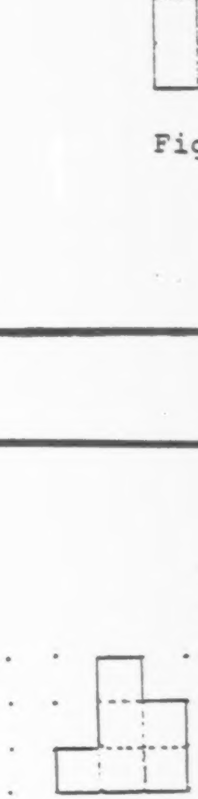


Fig. 1

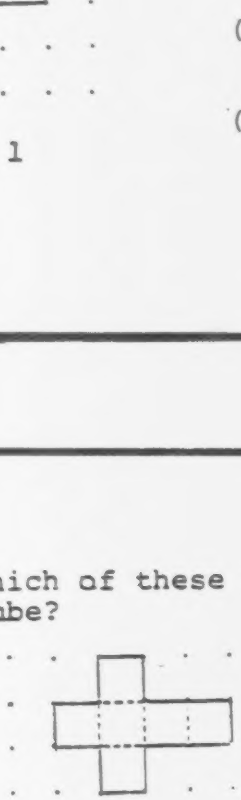


Fig. 2

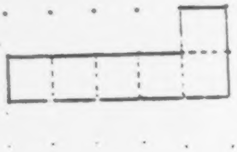


Fig. 3

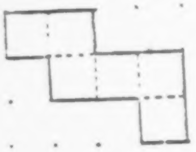
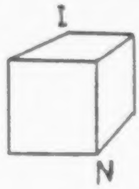


Fig. 4

Fig. 1 and fig. 2 (A)
Fig. 2 and fig. 3 (B)
Fig. 3 and fig. 4 (C)
Fig. 2 and fig. 4 (D)

How many different paths are there from I to N along 3 connecting edges of the cube?



(A) 3

(B) 4

(C) 6

(D) 8

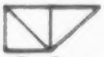
The figure  represents an arrangement of 3 triangular tiles. Which figure below contains the same arrangement?

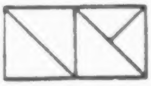
Fig. 1


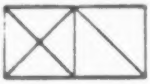
Fig. 2


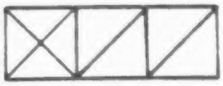
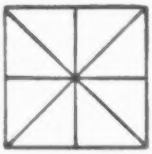
Fig. 3


Fig. 4


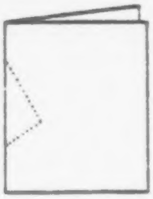
(A) Fig. 1

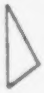
(B) Fig. 2

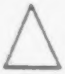
(C) Fig. 3

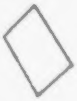
(D) Fig. 4

A sheet of paper has been folded, then cut along the dotted lines, as shown. When unfolded, the cut-out piece looks the same as



(A) 

(B) 

(C) 

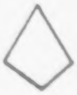
(D) 

Figure 1 represents a cube with faces labelled A,B,C,D,E, and F. In which case does Figure 2 represent a rotation image in the given cube?

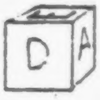
(A) 

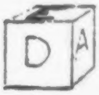
Fig. 1


Fig. 2

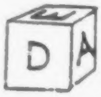
(C) 

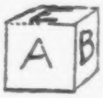
Fig. 1


Fig. 2

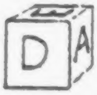
(B) 

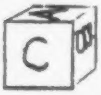
Fig. 1


Fig. 2

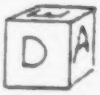
(D) 

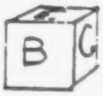
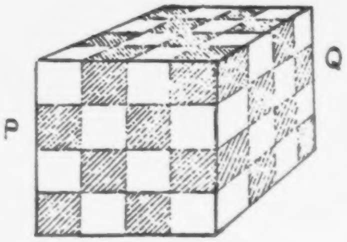
Fig. 1


Fig.2

A cube is made up of 64 blocks, alternating black (B) and white (W) as shown. The sequence of letters identifying the blocks along from P to Q is



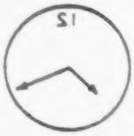
(A) B,B,B,B

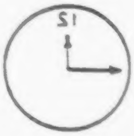
(B) B,W,W,B

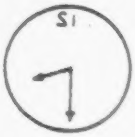
(C) B,B,W,W

(D) B,W,B,W

These diagrams illustrate images of clock faces in a mirror. Which image shows that the time is 3:30?

(A)


(C)


(B)


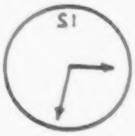
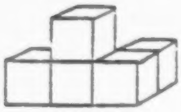
(D)


Figure 1 represents an object made from 5 cubes. In which case does Figure 2 represent a rotation image of the given object?

Fig. 1



(A)

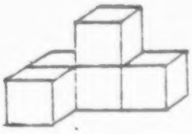


Fig. 2

(C)

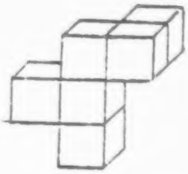


Fig. 2

(B)

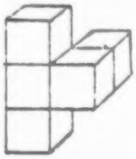


Fig. 2

(D)

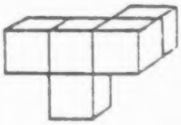
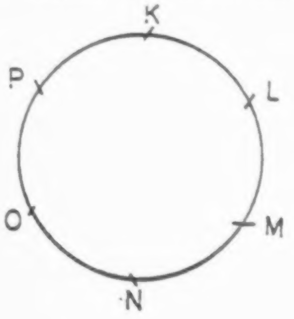


Fig. 2

Points K, L, M, N, O, P are equally spaced on the circle. Which figure cannot be drawn using some or all of these points as vertices?



(A) a six-pointed star

(B) a regular hexagon

(C) a rectangle

(D) a square

FIVE ANCHORS

Thomas C. O'Brien

We have just finished an exhilarating two-day seminar on "Toward the Twenty-First Century in Mathematics Education" as I write this, and while the other participants are contributing SEEDBED papers on what could/should be borne in mind as one looks forward to the twenty-first century, I propose to address some issues which, I think, are keeping us from moving ahead in mathematics education.

I have chosen "Five Anchors" for my title. "Anchor" is an ambiguous word. To some people it means "mainstay" or "root" or "that which keeps a ship safely in harbour." I have another connotation in mind: "that which forestalls movement, progress, change."

Anchor #1: Finger counting.

Some teachers discourage children's counting on their fingers. These teachers see mathematics as the automatic association of two numbers--say, 3 and 5--with a third number--say, 8--as mathematics.

There are several points to be made:

Such a view deprives children of relationships. What happens is that children see 3 + 5 = 8 as one issue and 3 + 6 = 9 as an entirely separate issue. But relationships are the heart of mathematics. To see a number as a bunch of clang responses is to throw mathematics out the window.

To be sure, children should know, without having to count it out, that 3 + 5 = 8. And an adult should know how, without hesitation, to drive a car or post a letter. But learning is not an all or nothing thing.

(Remember when you first learned to drive?) It does not happen instantly.

One aspect of our seminar dealt with the differing definitions of mathematical entities throughout history. Many mathematical ideas have had a dozen different definitions. And so do ideas, in general, change over time. Malaria was one thought certain to be caused by bad air. More important for the present discussion, a person's knowledge changes. Is your knowledge of politics, or New York, or the Ten Commandments the same as it was when you were ten years old?

Knowledge is a personal construction. Knowledge of number is a personal construction, not a given, once-and-for-all, static "fact." Knowledge of number has its origins in counting. In fact, much if not most of the present American elementary school math curriculum amounts to counting. (Multiplication of whole numbers is essentially rapid counting.)

Piaget has given a neat definition of thinking. "Thinking is interiorized action." When a child counts on his fingers and touches and moves objects when counting, (s)he is acting on those objects. Adults can count change or people or soda bottles without touching and moving (and finger tapping, head nodding, etc.). Those actions have become interiorized. To deprive children of action--such as that in counting on fingers and beadstrings and Unifix--is to deprive them of thinking.

But what, you say, does one do when a thirty-one year old person has to count 3 + 5 on his/her fingers? If we encourage finger counting, won't it persist? The answer is a developmental one: you and I once crawled before we walked. Few of us got stuck in the crawling stage. The fact is that those children who have been taught clang number associations are the ones who use their fingers (and who learn, besides, to become sneaky about doing so) while the ones who have constructed number through action use relationships (3 + 7 is one more than 3 + 6 so 3 + 7 = 10; 7 + 5 is "seven get to ten, go two more") are, in fact, the ones who are more efficient.

Here are some activities for teachers to learn about children's knowledge.

- Ask children to count a bunch of chips you have spread out at random. What's the range of strategies children use?
- Ask a child to show you five fingers. Then ask for six fingers. What's the status of this issue with your kids?
- Ask children to count out 20 chips. Then say, "Sorry, I goofed. I wanted 17." What do children do?
- Spread out a number of chips. Ask a child to count them. Then hide some chips, and ask the child to count what's showing and to figure out how many are hidden.
- Ask a child to use fingers on each hand to show six. "Can you do it another way? How many ways can it be done?"

Anchor #2: Hand calculators

At a recent teachers' meeting, the subject of hand calculators came up. Most teachers felt that hand calculators were inappropriate for the classroom. At best, teachers expressed some ambiguity about the issue. One teacher told a true horror story: children in her class could use hand calculators, but only to compute the standard alogrithms. In long division, she said, kids did every step in the usual way but they checked each step with a calculator. Holy cats, I thought. That's like using a word processor to produce script writing.

But the issue is more fundamental. The blind repetition of recipes has long been a part of mathematics education. To what end? Computational proficiency was a marketable skill in the 1910's. It is not now. The latest generation of computers, particularly the Cray supercomputer, can do 100 million long computations in one second.

One of the purposes of schools, it has been widely said, is to prepare children for society. Hand calculators are part of American society. They are not part of our school programs. Shouldn't part of the math education program be given to using hand calculators, testing their limits, comparing the characteristics (and logics) of various calculators?

The use of hand calculators can enrich a school program by freeing children of the drudgery of lengthy computations so that children can get on with some real investigations using number. In an elementary school ecology investigation, A. I. Weinzweig reported at our seminar, children in grade 4 were given the task: "How many trees were needed to produce the paper that is in our school right now?" An incredibly rich investigation took place, one which would have been unthinkable if computations had had to be done by hand. And what of the use of calculators to investigate mathematics? What are the possible answers to 3 + 4 * 5? Is it (3 + 4) * 5 which is 35 or 3 + (4 * 5) which is 23? The fact is that the issue is ambiguous and so we use the parens to clarify what is meant.

What other investigations are possible? How about differences between consecutive squares: 1, 4, 9, 16, . . . Is there a pattern? Does this pattern hold for big numbers? What printout comes for 1/9? What other combinations produce something like this? What's the pattern in the 9's table, 9, 18, 27, etc. Does that hold when you get up into the thousands? Any pattern for 8's? 7's?, etc. Any pattern for the 99's table? Why is a number such as 397,397 (or any abc,abc) always divisible by 13?

Finally, with the widespread availability of computers--it's predicted that the home computer will be as commonplace in the American home in 1985 as television sets are now--what's the role of algorithms? A big role. Computer programs are algorithms. How about classroom use of hand calculators (until computers become available in schools, something not too far off?) for kids to construct alogrithms? What is square root for example?

Anchor #3: Competition

I take the liberty of reproducing the text of the page from Jules Henry's Man Against Culture which Dick Tahta brought to our attention during the Tenth Annual Teachers' Center Project Summer Course.

> Boris had trouble reducing '12/16 to the lowest terms and could only get as far as '6/8'. The teacher asked him quietly if that was as far as he could reduce it. She suggested he 'think'. Much heaving up and down and waving of hands by the other children, all frantic to correct him. Boris pretty unhappy, probably mentally paralyzed. The teacher, quiet, patient, ignores the others and concentrates with look and voice on Boris. She says, 'Is there a bigger number than two you can divide into the parts of the fraction?' After a minute or two, she becomes more urgent, but there is no response from Boris. She then turns to the class and says, 'Well, who can tell Boris what the number is?' A forest of hands appears, and the teacher calls on Peggy. Peggy says that four may be divided into the numerator and the denominator.
>
> Thus Boris' failure has made it possible for Peggy to succeed; his depression is the price of her exhilaration; his misery the occasion for her rejoicing. This is the standard condition of the American elementary school, and is why so many of us feel a contraction of the heart even if someone we never knew succeeds merely at garnering plankton in the Thames: because so often somebody's success has been bought at the cost of our failure. To a Zuni, Hopi, or Dakota Indian, Peggy's performance would seem cruel beyond belief, for competition, the wringing of success from somebody's failure, is a form of torture foreign to those noncompetitive redskins. Yet Peggy's action seems natural to us; and so it is. How else would you run our world? And since all but the brightest children have the constant experience that others succeed at their expense, they cannot but develop an inherent tendency to hate--to hate the success of others, to hate others who are successful, and to be determined to prevent it. Along with this, naturally, goes the hope that others will fail. This hatred masquerades under the euphemistic name of 'envy'.

Dare I add to what Jules Henry has said? Yes, but you might want to read Henry once again. Then wait a bit so that Boris's story can sink in.

There's no question but that American education relies heavily upon competition for motivation. In doing so, we tap only a small rivulet when there's a wellspring nearby.

Competition is one of many aspects which engage a basic, built-in, life-sustaining quality that we have: the tendency to seek equilibrium and, having achieved equilibrium, to set up disequilibrium so that it, too, can be reduced. (Ever watch a two-year old, or a crossword puzzle solver, or a mountain climber? Ever watch a Boston automobile driver?)

A general term for that which causes disequilibrium is dissonance. Dissonance is the mismatch between where we are and where we want/need to be (or where the situation calls for us to be).

Competition is one aspect of dissonance. And there's no doubt that competition dissonance works for some. As with all dissonance, though, there's a three step process: 1. Encounter dissonance. 2. Decide to engage or ignore it. 3. If engage, resolve. The problem is that many children--some slow, some very bright--decide not to enter competition. For the slow the gap is too large. For the very bright the issue is too trivial.

Dissonance encompasses much more than competition and dissonance in general is at the heart of the learning situation.

A major aspect of dissonance is novelty. A situation which is moderately novel is often irresistible. (The degree of novelty, of course, depends on the mismatch between the person and the situation.) To what extent does American advertising capitalize on novelty? To what extent does popular music employ novelty? To what extent does American industry build novelty into products?

What happens when novelty is lost?

The problem, it seems, is to allow novelty to play a role in education without becoming an end in itself. This is not too difficult when one considers the role of novelty in a child's spontaneous learning.

Other aspects of dissonance, ubiquitous in real-life situations and intrinsic to the development and learning, are risk, uncertainty, and incongruity. To engage and resolve these aspects of mismatch--and to create them just for the sake of resolving them--is built into us all. Should these aspects of dissonance be ignored in education?

But a third aspect of dissonance is the human tendency to move past one's present state, to grow, to be the best one can be. This tendency is, in fact, competition but it is competition with *one's self*. This sort of competition is far more enriching than competition for external ephemera--grades, gold stars, teacher's praise, and the like.

Anchor #4: The tyranny of print

For some reason, Americans think that a thing really, really exists when it's in print. It's just the opposite, I hear, with certain other cultures. For the American Indian, a thing loses its life, its spirit, when it goes into print. And the British educators I have worked with are often loath to commit their curriculum to print. "Once we write it down," they say, "it'll lose its life, its spontaneity. It'll be fixed forever and what we do with kids will be frozen."

There are two aspects of print that dominate American education: paper-and-pencil work and textbooks (while children learn) and formal tests (to assess children's learning).

About textbooks and paper-and-pencil materials in general:

If action is the heart of knowing and if knowledge is interiorized action, can we bring about knowing by confining children's actions to just one kind:

writing things on paper? Writing things down is a summary--and an important summary--of what was done. And it is an important external memory of what was done. But how about doing--sorting, measuring, counting, constructing relationships--in the first place?

It is commonplace that a textbook be, by and large, the math curriculum for a classroom. The same textbook for every child. Try some of the activities in Anchor #1 above and tell me whether one of anything fits all your kids.

Shirley Casey and I have just finished a lengthy investigation of 3-4-5-year olds on the Hidden Chip task. The range of strategies--from wild guessing to extraordinarily rich and complex constructions--was enormous. If this is the case with 3-5 year olds, what of the range of older kids? Would you give one size shoe to all the children in your class?

Most large publishers will not publish anything but a basal text in mathematics. The same text for each child. (Oh, yes: ancillary materials, too, but they must be intimately related to and cross-referenced with the text.) The reason, I must say, has nothing to do with children's learning. It has to do with profits.

About testing:

The external "standardized" test is a mainstay in American education. Some comments:

I won't repeat the comment about profits. Too obvious.

One of the things we do is to administer tests at the start of the school year. Fresh from summer vacation, children sit for as much as two weeks of group testing. The tests are then sent off for computer scoring and returned somewhere about mid-November. At this point, more often than not, the results--numbers--are stored in an administrator's file. Does this make sense in terms of a) capitalizing on children's freshness at the start of the school year? b) costs in lost time? c) costs in terms of benefit to children, to curriculum revision, to teachers' knowledge of children?

It seems to me that the important thing about assessment is that it can help teachers to help children. My strong feeling is that paper-and-pencil tests are extremely successful in keeping teachers from looking at children. As Senator Sam Ervin said during the Watergate inquiry, "You can't question a piece of paper."

There are several levels at which one can look at children's knowledge:

One can assess the results of a child's thinking. This is the most common form of assessment. It is generally done on a right/wrong basis. And it is only a small step in the direction of finding out about where a child stands. Knowing whether a response is right or wrong is very little help to the teacher as far as future action is concerned.

(I have told this story many times: Teacher says to five-year old, "What's nine plus nine?" Five-year old says, "nineteen." Teacher says, "How'd you get that?" Child says, "Easy, ten plus ten is twenty, so nine plus nine is nineteen." Need I say more about the "right vs. wrong" sort of assessment?)

It is actions which produce results (i.e., "answers") and it is actions that we should look at. This is where paper-and-pencil tests are of no help unless one looks at children's answers and then tries to work backwards to guess how the result was obtained. Perhaps better, one can look at children's thoughts and actions as they take place, rather than guessing backwards.

And, at an even deeper level, hypotheses inform actions. What is the child thinking about when he performs action such-and-such. What is his/her hypothesis? What are the general mind currents that guide the actions which produce the answer? It is at this level that paper-and-pencil tests shed no light at all. And this is the most important inquiry of all if we are to help children build a fabric of knowledge rather than a bunch of unconnected bits and tricks.

Anchor #5: Exercises

The fifth anchor is the dominance of exercises. In physical education, an exercise is a series of actions routinely and repetitively performed. In math classrooms we have children doing things like this page after page and day after day:

$$\begin{array}{r}34\\ \underline{x47}\end{array} \qquad 3 + \underline{\quad} = 12 \qquad 4 + 8 = 1 + \underline{\quad} \qquad 27\,\overline{)\,390.42} \qquad 3/7 + 4/5$$

And we have page after page of computation disguised by words, such as:

> Terry the Tiger jumped across a river that was 645 feet wide. Then he jumped 214 feet. How far in all?
>
> Terry the Tiger drank 9 gallons of milk and 8 gallons of water. How much did he drink?
>
> Terry the Tiger weighs 462 pounds. He ate a big meal and gained 42 pounds. How much does he weigh now?

The point is that I think there are three levels of "phys ed" in mathematics. The level at which we are pretty much stuck is the level of exercises, routine actions mechanically and repetitively performed. Exercises have one goal and usually one method of achieving that goal. No self-respecting mathematician would dignify this stuff with the word mathematics.

A second level is problem solving. Here there's a goal and the task is to construct actions and to organize them to achieve that goal. In much problem solving, in fact, there may be several (or no) sets of organized actions which achieve the goal. Several examples of problems, which differ quite patently (for most readers) from exercises, are the following:[1]

> Break the code if each letter represents a different digit from 0-9.

$$\begin{array}{r}\text{LOTS}\\ \underline{+\text{AFUN}}\\ \text{CCCC}\end{array}$$

> How many ways are there to get 7 litres of water, given a spigot, an 8 litre container and a 3 litre container?
>
> How many people in the United States have the same birthday as you?

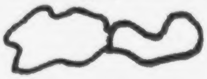
A third level, where almost no time is spent in American mathematics education, is the investigation. One gross distinction among exercise, problem solving and investigation is the time it takes to complete the task. An exercise is usually completed pretty quickly, while a problem takes a longer amount of time and an investigation even longer. Pretty indistinct.

A second difference between exercises on one hand and problems and investigations on the other is that exercises usually call for the practice of an organization handed over full-fledged to children, while a problem and an investigation call for one child to be an agent, to organize and coordinate the reality with which (s)he is dealing. And another gross distinction among exercises, problems and investigations is that with exercises neither the goal nor the means belongs to the child. (S)he is given the task and the means to fulfill it. With a problem, the goal is often provided but the solution is not. And with investigations both goal and means belong to the learner.

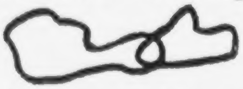
But this is still a bit sloppy and the careful reader can surely see overlap. The task remains to distinguish problems from investigations. Perhaps the best answer is that one can come back to a problem and find new and more parsimonious solutions whereas investigation admits new goals as well as new solutions. An investigation is one which a person can return to throughout one's life. Here are some investigations that we have found to be worthwhile:

1. Start with your LUCKY NUMBER and double it. Keep going until you pass 100. Then start over again and double untill you pass 1000. Write down three true things.

2. Two loops can have touching points, like this:



Two loops can also have crossing points, like this:



Set a challenge for yourself such as, "I'm going to try for three crossing points," or "I'm going to get two figures with exactly three touching points." Draw some loop figures and try as many challenges as you can. Then write three discoveries.

3. Find as many patterns as you can.

0	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

Concluding Comment

Perhaps you can think of other anchors yourself. The ones I have listed seemed particularly important at the time I wrote this. One of the most important aspects of resolving a difficulty--any difficulty--is to identify it. Given that some decent anchors to progress have been identified--and given that you have added to the list--let's roll up our sleeves and do something about them.

NOTES

1. This activity and the ones which follow come from Solve It Books 1-5 (Chicago: Educational Teaching Aids), written by T.C. O'Brien.

COMPUTERS AND THE FUTURE

Robert S. Hoecke

What I would like to do is to reflect first on my personal experiences and to give you a little background. I was educated as an engineer because my father was an engineer. Engineering never took to me (or I never took to engineering).

I started with computers in 1951 and have been working with them ever since. The first computer we had was a 2K machine with paper tape output called a "Bendix G14". It was such a big machine that it took four tons of air conditioning just to keep it cool. Now in 1982, we have moved down to minis and little pocket calculators that make those early machines look "cow-like".

I really have two concerns in the area of math education at the primary and secondary levels. I've been a production manager, so I understand that side of the world (i.e. the real world). My primary concern is a general requirement for educated, useful people (if you can combine those two and I think you should). It's something called computer literacy. Computer literacy requires mathematical literacy. I'm amazed that we're trying to achieve computer literacy without mathematical literacy.

We get students at the university level who come from a depraved (not deprived, but depraved) primary and secondary educational system, a system that teaches fear of mathematics--rather than utility of mathematics. I think we must teach mathematical literacy. These best of today's students can take an equation and even manipulate it with a calculator, but they have no feeling for what they are doing. "That's the way we get the answer", is what they say. And you know, mathematics examinations are geared this way. The professor changes the numbers on the exam from what he did in class. And the student answers the test items correctly and the professor tells him how good he is to regurgitate the process he taught. This isn't literacy.

I attempted something that I must admit was my most classic failure. I tried to administer a mathematics literacy examination to our graduate students before I took them in on a course. The engineers failed and the mathematicians failed. That is the issue. Their experiences have been a depraved "get the answer" education.

One answer to this has to be introduce finite mathematics. Remember 15 years ago when some schools decided to do finite mathematics? We can technically demonstrate that it is a superior way to present mathematics. No question about it, it's a way to produce mathematical literacy. These kids can understand models, they understand probability and logic, they understand the whole concept of mathematics. It failed because the teachers were not taught that way themselves. They knew only how to transmit procedures, to say, "answers are in the textbook" and "this is the way to do it". It was clearly a superior way to present concepts, but the teachers (through no fault of their own) were not trained to teach with a clear command of the concept. Again, this is not mathematical literacy. They rejected it because they didn't understand it. They felt that they didn't get as much done during class time. I understand their concern. It's one of my main concerns.

There's a second real issue concerned with the use of computers. I, like a lot of others, have used "PLATO"--this is one of the many applications within CAI (Computer Assisted Instruction). I look very carefully at a subject and see what parts of it can be taught from a textbook. CAI is at least 100% better than a textbook to teach any material that could be taught from a book. I honestly believe that. I have seen it operate. The student is always in a supportive environment. There's never a condemning environment. And the computer is almost infinitely patient. For example, if the child has problems dealing with the 7's, CAI automatically takes him into an exercise to reinforce the 7's. It's private. Kids are not being openly compared to their classmates. No teacher embarrasses a child by saying, "That's good, Johnny, but you need more work on your 7's."

As to the future, reflecting on both of these concerns, mathematical literacy and computers (CAI), up to now we have had two variables in our classrooms--behavior and learning. We've attempted to hold behavior constant. We have said, "In order to learn this, you must behave in a certain way. Don't ask questions because this is the only acceptable behavior." If the teacher is teaching and a little discipline problem comes up, frequently, the teaching/learning stops and the teacher does whatever is necessary to restore the desired behavior. The result is that many teachers spend inordinate amounts of time teaching very inefficiently. Teaching from textbooks on a human interface is very inefficient and is not always totally successful. Over the next five years--the next ten years at the outside--I think that we will move toward replacing teacher/textbook presentations with learning from computers because it's better, it's more efficient, and the students have a 100% supportive environment.

When I first started reading and thinking on this issue, I wasn't sure of the feasibility of widespread computer usage. I was thinking in terms of computers that were so ungainly that one had to load them onto a trailer in order to move them. They could never get into the home. But now that's possible. (The Sinclair computer can be bought for about $200 and carried under your arm. It's a home computer, a very usable one.) By connecting in with communications systems (now the telephone lines, but very soon cable TV links) we'll have nearly limitless facilities available in the home. Bound volume libraries are very nearly extinct using this form of communication linkage. Virtually all law search today is done by computers: you don't send a law clerk or law librarian to look in the books--the chances are that he/she will miss the most significant case--you go to the computer, to the data bank. Drugstores are advertising that when you get a prescription filled they will check for counter-indications with all other medications you might be taking. And so on, as we move into the 21st century.

I think, however, that we will equip classrooms with computers. In school district after school district we are seeing this take place (Staunton, IL, for example, not a rich district by any means, has put in computers and is moving rapidly to integrate computer usage from K-12. And they are late!)

Working with computers is naturally attractive. Programs of all types are widely available from games to business to education. For example, the computer company that I do business with has a small, $19.95 typing tutorial program that in one extensive week of work can get children or adults typing in excess of 60 wpm with no more than one error. The same sort of programs exist for arithmetic skills and in a lot of other areas as well. I've used them with my own kids.

This is the direction in which we are moving to replace a lot of textbook work with computers and through the flexibility of computers, to teach additional worthwhile things.

If we then do not expand the depth or breadth of topics, what will we do with the extra time that will be freed? Are we going to pack in more materials and increase the requirements? I think not. I think that we're going to take the other half of the world--the behavioral world--and spend some very legitimate time addressing human issues. Too many people lack the ability to effectively communicate. They lack the ability to work in crisis situations without jumping up and down and pounding on the tables (or someone's head). My thought is that as we replace some of the teacher/student relationship in some of the three R's, we will start spending more time on behavioral aspects. The key to it is the technology of the computer. It can do certain things more effectively than people can do, and thus free us to spend time on those things that we have not had time nor energy to address. We have forced kids to behave in a singular fashion because it has been easier to teach in a uniformly disciplined situation. That will no longer be true with computers. Let the computers do the machine-like things and let humans do the human things.

Selected Questions and Answers

O'Brien: What do you see happening by 1990 with respect to saturation of the market?

Hoecke: By 1990, computers will pervade our society. They will be as common as TV sets are now. And they will be much less expensive. For example, you can get a fairly decent microcomputer now for around $300--and those computers can be linked to other computers via phone or TV cable.

O'Brien: I heard on KMOX radio a couple of weeks ago the notion that by the year 2000, 50 percent of the jobs in the US will be jobs which are presently unknown. Does that make sense?

Hoecke: A few years ago I worked for DuPont. They said that 50 percent of their profits that year came from products that had not existed ten years before. So it's not hard for me to accommodate to the notion that half of the jobs that will exist in twenty years are unknown today.

It won't be easy to provide for that kind of change. The pressure on educators is even greater than in the past. This goes back to the thing I talked about earlier. It's not "Can you use mathematics?" but rather it's the innate sense of knowing how to use mathematics as a tool to help you do your job better.

O'Brien: What sort of attitudes and abilities does a person need to deal with 1990 and beyond?

Hoecke: For one thing, an openness for new information in a changing world and a willingness to reach out to the other guy.

I'd like to refer to a statement Bronowski made: "The greatest power of all of civilization and its growth is the fact that having done something once, the next time we encounter it we wish to do it better." I think the same thing applies to learning. In a supportive environment, the only limit to education is one's imagination.

O'Brien: How can computers alleviate the boringness of schools?

Hoecke: If a computer doesn't do what you want, you can tell it "goodbye". Insults aren't given. The learner doesn't have to do the same things over and over. And if available programs aren't achieving their purposes, new and better programs can and will be written. The important facts are these: the learning environment is always supportive, is always impartial in judgment, and is always challenging to the student at each stage of his/her learning. Learning is self-paced. The depth and breadth beyond the requirements are available for those who wish the challenge. Beneath requirements, learning is routed to areas of weakness and repeated patiently until mastery without student embarrassment or condemnation is achieved.

O'Brien: How are computers going to change the school curriculum?

Hoecke: When computers are built into every school desk, a skill like typing will become a necessary first grade skill. And people will learn composition using a word processor so all the rewriting of papers will be a thing of the past (And the finished product can be transferred from one computer to another via phone lines.)

We used to say a person was educated if he could read, write and cipher. I'm not sure what "educated" means now. (We had to manually extract cube roots.) I do know that I want to maximize the individuals, to help them get what they need to know in order to make valid contributions and decisions. I's like to be able to set an individual into motion and let him go as far as he can go or wants to go.

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